ETT-311 SIGEx Solutions Manual

[for Instructor's use only]

Signals & Systems Experiments with Emona SIGEx
Volume 1

EMONA

tims

TELECOMMUNICATIONS MODELING SYSTEM

.**Signals & Systems Experiments with Emona SIGEx**

Instructors Solutions Manual

Volume S1 - Fundamentals of Signals & Systems

Authors: Robert Radzyner PhD
Carlo Manfredini B.E., B.F.A.

Editor: Carlo Manfredini

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Emona Instruments Pty Ltd,
78 Parramatta Road
Camperdown NSW 2050
AUSTRALIA.

web: www.emona-tims.com
telephone: +61-2-9519-3933
fax: +61-2-9550-1378

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Note to Instructors

The EMONA SIGEx Lab Manual contains 3 types of questions.

(i) **Pre-Lab preparation** questions, which review the theoretical principles a student may need, to get the most out of each experiment.

(ii) **Experiment** questions, in response to findings within the experiment itself, as the student carries-out the experiment.

(iii) **Tutorial** questions, which are suggested **optional** questions to further reinforce the theoretical principles covered in the experiment.

This manual is provided as a **convenient guide,** for instructor's use only. It offers suggested answers to the various questions posed in the SIGEx Lab Manual. Due to intentional gain and phase variations between different SIGEx boards, it should be understood that each student's responses, as measured, may differ by more than +/-10% with respect to the answers presented in this manual.

Instructors may also prefer to formulate their own answers to theoretical questions, and these may differ from those presented in this manual.

The SIGEx Lab Manual and Instructors Manual is **not a replacement for a textbook.** It is primarily aimed at guiding students to implement their learnings from formal lectures, in a hands-on, experiential manner.

Students will almost certainly **learn more from their mistakes** and misapprehensions, than they will from completing the experiments without incident. Taking time to sort out unexpected results will be of great benefit to their learning process.

The SIGEx board is **not calibrated**. In fact, it is considered a virtue of the **hands-ons modelling approach** that circuit responses between boards may differ slightly. This will result in slightly different responses from the various circuit blocks. Adjacent students will therefore need to pay attention to their own measurements rather than copying the results of others.

Answers to theTutorial questions are not provided, as these questions are suggested as optional work, if time permits. It is left up to the individual instructors to provide guidance in lectures about these questions.

We hope that your students enjoy working with the EMONA SIGEx board and welcome your comments via email at any time.

Best regards,

Carlo Manfredini

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EMONA SIGEx Instructors Lab Manual Volume 1

For Instructors use only

Contents

Name: ______________________________

Class: ______________________________

X3 - Special signals - characteristics and applications

SIGEX

Question 1
What is the minimum interval of the SEQUENCE GENERATOR data ?

1 ms

Question 2
Describe the signal transitions for both outputs:

BBLPF, has some overshoot then settles, whereas the TLPF has several cycles of overshoot before settling.

Table 1: transition times for sequence data

Range (%)	BLPF@1kHz (us)	TLPF@1kHz (us)	BLPF@1.5kHz (us)	TLPF@1.5kHz (us)
10-90 rising	220	70	220	75
10-90 falling	220	70	220	75
1-99 rising	360	114	350	114
1-99 falling	360	114	350	114

Question 3
Describe the signal transitions for both outputs:

Above 3500 Hz the BLPF signal no longer transitions completely between states for 0-1 patterns. The channel places a limit on the transition rate.

Table 2: transition times for step input

Range (%)	BLPF (us)	TLPF (us)	RCLP (us)
10-90 rising	210	70	2120
10-90 falling	210	70	2120

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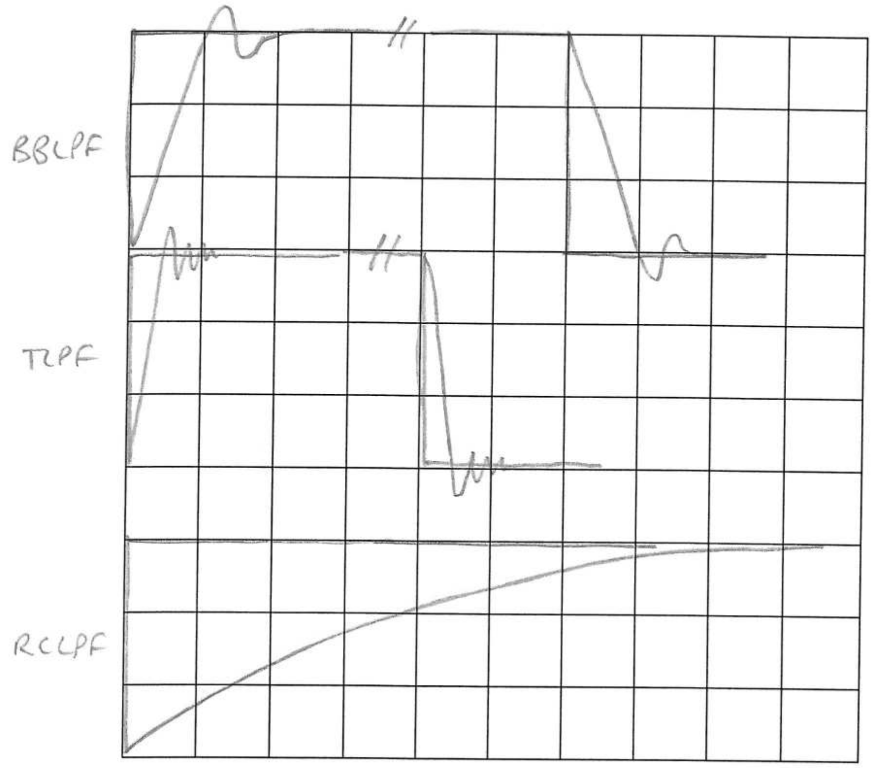


Experiment X 3 : Graph 1

Graph 1: step response waveforms

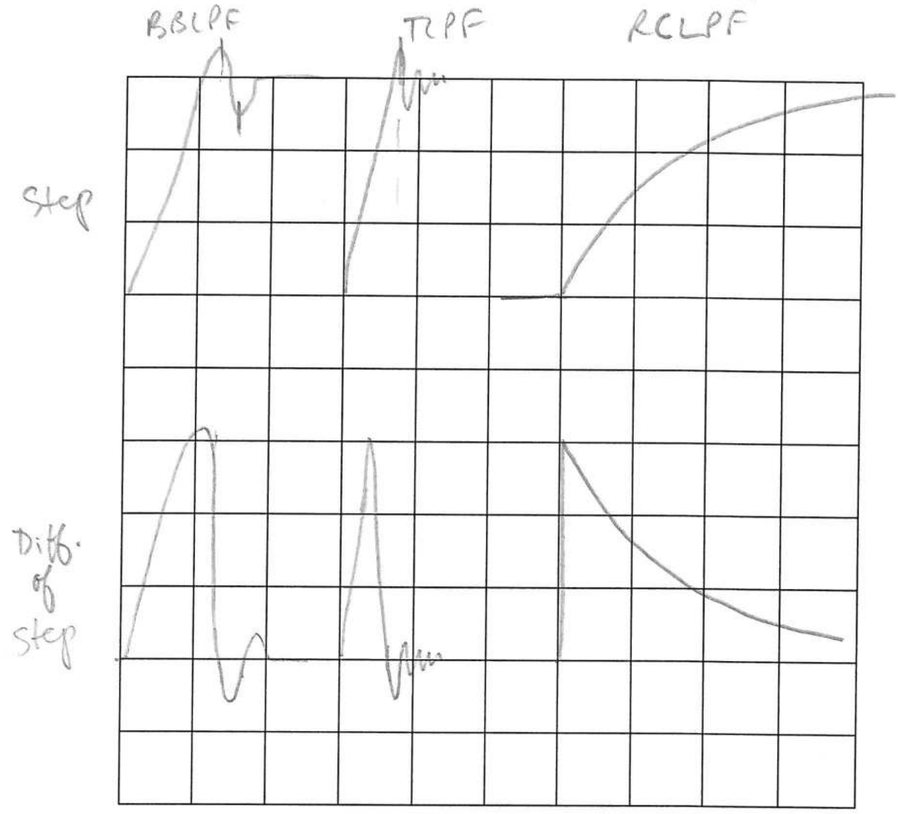
Question 4
Describe what happens when you reach 10% and 5% duty cycle ?

The amplitude of the BLPF output begins to reduce.

The TLPF output begins to stop ringing and have no "flat top" at all.

Table 3: pulse response readings

	BLPF	TLPF	RCLPF
Duty cycle "demarcation" value	0.1	0.04	-
Calculated pulse width (us)	400	160	~3000
% of step response	200	200	150
Period of oscillations (us)	480	140	-



Experiment X3 : Graph 2

Graph 2: differentiations of step response waveforms

Table 4: amplitude vs frequency readings

Frequency (Hz)	BLPF (Vpp)	TLPF(Vpp)	RCLPF(vpp)
100	3.45	3.4	3.4
500	3.45	3.4	1.3
1000	3.26	3.4	0.68
1500	2.85	3.4	0.48
2000	2.05	3.36	0.35
4000	0.16	2.9	0.2
6000	0.1	3.14	0.13
8000	<< 0.1	1.72	< 0.13
10000	<< 0.1	< 0.1	< 0.13

Question 5
What frequency would a matching sinewave have ?

Its period would be twice the step response time ie: BBLPF=420us:2380 Hz.

TLPF=140us:7142Hz. RCLPF=4000us:250Hz

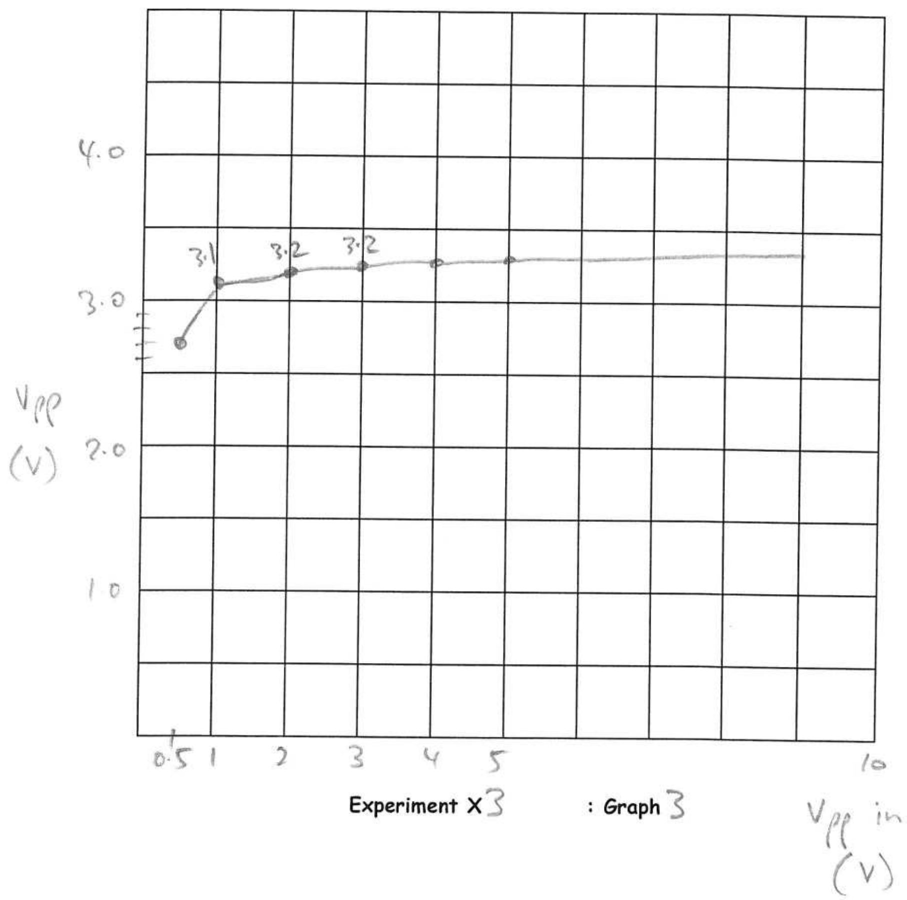
Question 6
Describe what happens to the frequency response plotted on the SFP at this frequency ?

The response starts to drop off at that frequency, down to approx. 0.7 times initial value

Question 7
What was the mechanism described earlier ?

The incoming signal doesn't have enough time to transition between levels before changing

direction, because the SUI's "inertia" (resistance to change) is slowing it down.



Graph 3: CLIPPER input and output readings

Next we use the CLIPPER as a primitive digital detector.

Question 8
How does this setup compare to the previous findings without a LIMITER ?

The LIMITER enables the recovery of signals at a much higher rate then without.

Name: ______________________________________

Class: ______________________________________

X4 - Systems: Linear and non-linear

SIGEx

Experiment 4 – Systems – Linear & Nonlinear

Question 1
Write down a formula to express the square of a sinusoid in terms of a double angle argument.

$[A\sin(wt)]^2 = \frac{1}{2}A - \frac{1}{2}.A\cos(2wt)$

Question 2
What is the meaning of differential linearity ?

A constant relation between the change in the output and input.

Question 3
How would you apply these formulas in testing systems for linearity in this Lab ? How many replicas of the system are needed for the additivity test ?

Implement the formulas with models using a module as a S.U.I

At least 2, but 3 for simulataneous testing.

Table 1

Input amplitude (Vpp)	LIMITER amplitude (Vpp)	RECTIFIER amplitude (Vpp)
1	3.2	0.42
2	3.2	1.28
3	3.2	2.2
4	3.2	3.2
5	3.2	4.1
6	3.2	5.0
7	3.2	6.0
10	3.2	9

Question 4
Does this system (CLIPPER) satisfy the scaling test for linearity? Show your reasoning.

No. The output does not follow the input proportionally.

Question 5
Does this system (RECTIFIER) satisfy the scaling test for linearity? Show your reasoning.

Yes. It does follow the input amplitude proportionally, for voltages above 4Vpp

Table 2

Input amplitude (Vpp)	MULTIPLIER amplitude (Vpp)
1	0.29
2	1.1
3	2.5
4	4.37
5	6.88
6	9.8

Question 6
Does this system (MULTIPLIER) satisfy the scaling test for linearity? Show your reasoning.

No. The output does not follow the input proportionally.

Squaring is a quadratic relation.

Table 3

Input DC voltage (V)	VCO output frequency (Hz)
-3	869
-2	1235
-1	1610
0	1970
1	2320
2	2700
3	3210

Question 7
Is the VCO a linear system ? Explain your reasoning.

Freq change varies proportionally with input voltage change

Question 8
What applications could the VCO with varying output frequency be used for ?

The input could be a message which varies an VCO output RF freq for transmission ie FM

Question 9
What is the formula for the INTEGRATOR output ?

Out(t) = k.integ (DCdifference) dt

K=10V/0.5ms/1.6V = 12,500 /s

Question 10
What are the formulae for the other INTEGRATOR rate settings ?

Out(t) = k.INTEG(+-DC dt)

INTEG DIPS = DW:DW saturates...rate too high for this frequency

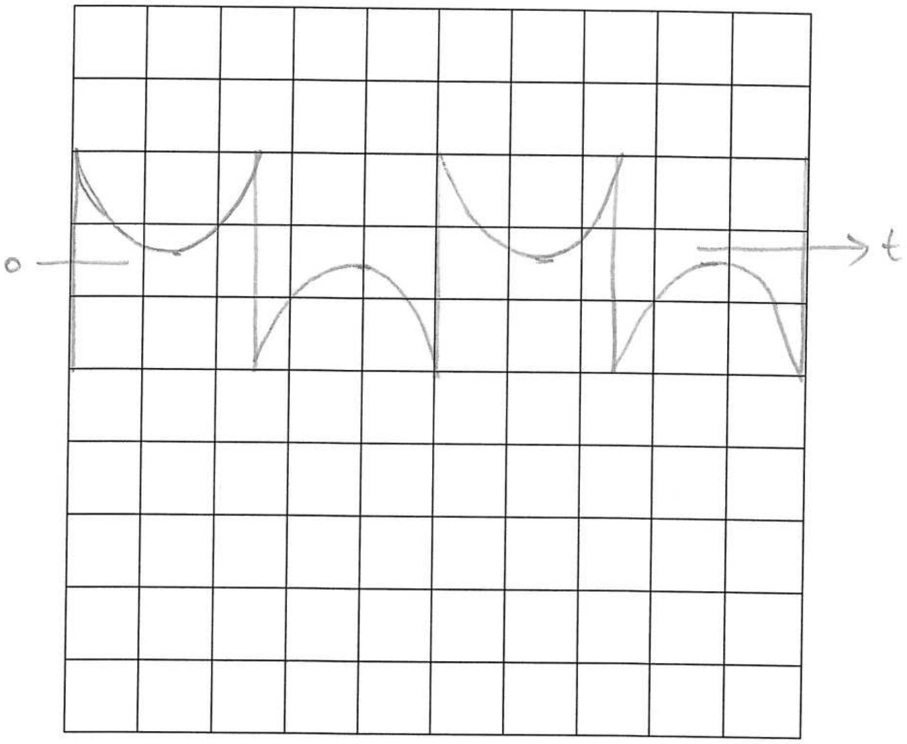
Question 11
Use the value of the b2 gain, and INTEGRATOR constant you measured above to determine the time constant of the exponential responses. Compare this with the value obtained from your measurement.

B2=-1, k=12,500

Time constant = 1/12,500 = 80us

Question 12
Write a differential equation for this first-order feedback system. Assume initial conditions are zero. Show that with a sinusoidal function of time as input, the output is also sinusoidal. Show that this also happens when the input is a complex exponential. Which special property of complex exponential functions provides the key?

y'(t) – b2y(t) = u(t); where b2=-1



Experiment X 4 : Graph 1

Graph 1: additivity signals

Question 13

Does the outcome indicate that the linearity conditions have been met for these two test inputs?

Yes

Question 14
Does the outcome during variation indicate that the linearity conditions are still maintained for these two test inputs?

Yes

Input frequency (Hz)	Square wave output (V)	Sine wave output (V)
100	1.73 V pk	1.78 V pk
300	1.73	1.78
600	1.82 average	1.78
900	1.94	1.64
1200	1.94	1.55
1500	1.82	1.42
1800	1.55	1.2
2100	1.17	0.92
2400	0.79	0.63
2700	0.5	0.41

Question 15
How are you able to use the square wave for this test ?

No. The output has ripple which varies with frequency.

Name: ______________________________

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X5 - Unraveling convolution

Experiment 5 – Unraveling Convolution

Question 1
Describe a procedure for confirming the GAIN at each tap ?

Remove leads to the B ADDERs leaving only 1 of 3 attached and view the output pulse height.

Question 2
Display the delay line input signal (i.e. at the first z^{-1} block input) and the ADDER output signal. Measure and record the amplitude of each pulse in the output sequence.

1V in, 0.3, followed by 0.5, followed by -0.2V pulses.

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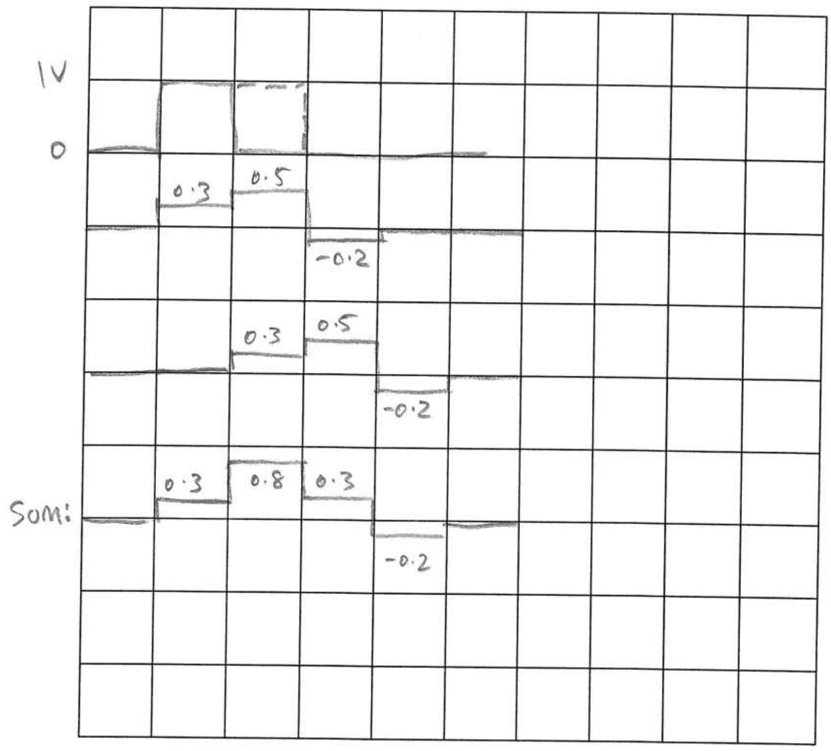


Experiment X 5 : Graph 1

Graph 1: unit pulse pair summation

Question 3

What is meant by “superposition”. Discuss how this exercise above relates to superposition and the “additivity” principle.

Treating parts of an input individually, and taking the sum of the outputs of the parts as

the output of the whole, as per the additivity principle.

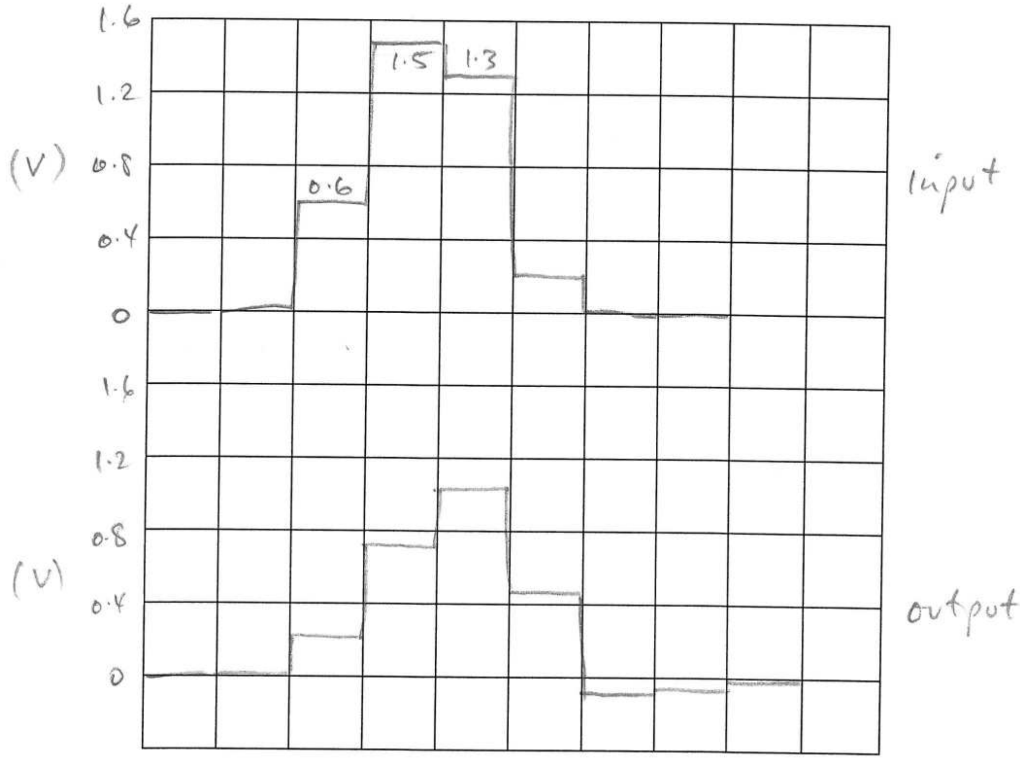
Question 4
What do you expect to see if this exercise were expanded to two or more contiguous pulses ? Explain.

A longer output pulse.

Question 5
Note the amplitude of the half wave rectified sine and explain why its amplitude is reduced relative to the input ?

The RECTIFIER is a real circuit, not an “ideal” device, and hence has a forward voltage drop

Of approx 0.5V



Experiment X 5 : Graph 2

Graph 2: inputs and sampled outputs

Question 6
How does this process relate to the principle of "superposition" ?

Addition of each component pulse into the complete output sum is using superposition.

Question 7
Write down the formula for y(2) and y(1) ? Discuss any unexpected differences .

$y(1) = b_0 .x(1) + b_1 .x(0) + b_2 .x(-1)$

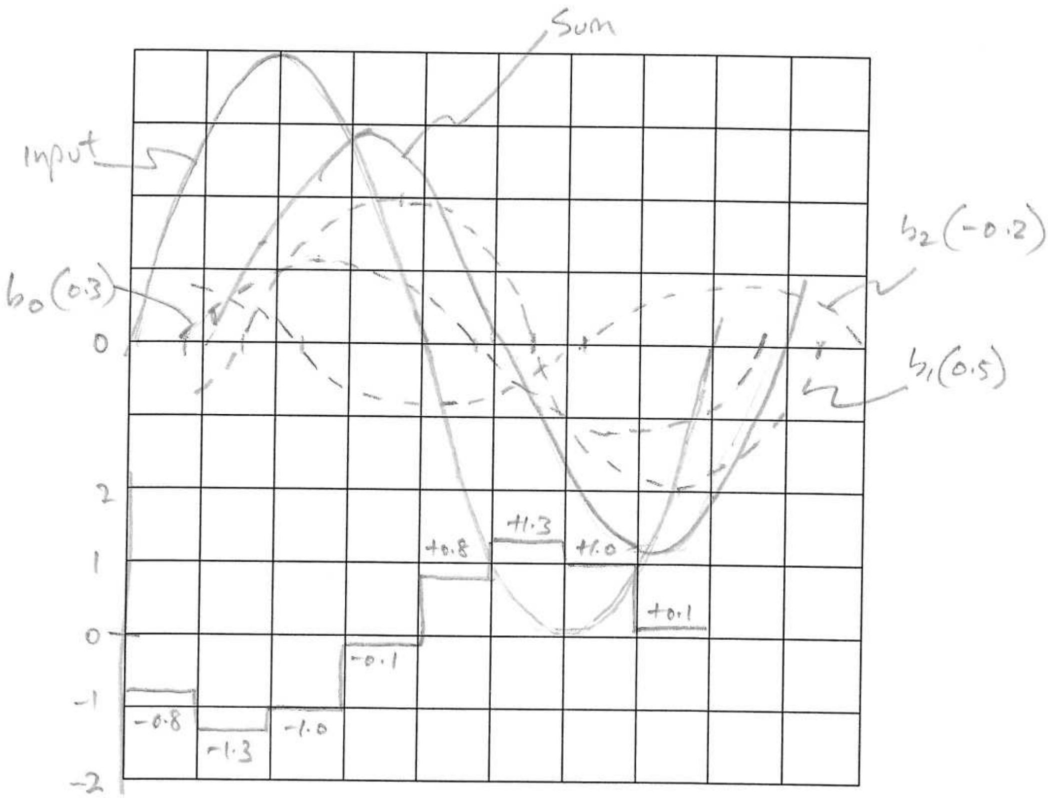
$y(2) = b_0 .x(2) + b_1 .x(1) + b_2 .x(0)$

Question 8
Explain why this term is reversed and what does this mean ?

It represents time reversal, and equates to the values being processed in reverse order.

Question 9
What is a common label for this response ?

3-point moving average (MAV)



Experiment X5 : Graph 3

Graph 3: sinewave input signals

Question 10
Show that the formula remains valid.

Question 11

Show your working for the sum of squares analysis ?

8 samples are :-0.8, -1.3, -1.0, -0.1, +0.8, +1.3, +1.0, +0.1. Pairs are [-0.8, -1], [-1.3, -0.1], [-1, 0.8] [-0.1,1.3], [0.8,1],[1.3,0.1].SS of pairs within 4% of each other

Question 12

Why is the outcome obtained above described as filtering?

The system passes frequencies selectively, hence it "filters" some and not others.

Name: ____________________________________

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X6 - Integration, convolution, correlation & matched filters

SIGEX

Experiment 6 – Integration, convolution, correlation and matched filters

Pre-requisite work

Question 1
For both a maximal length PRBS, of 31 and 63 bit length, calculate the ACF function values for all possible positions.

ACF of m-sequence of period N = N for k=0; -1 for 1 <= k <= N-1

Where k = delay index. Use N = 31 & 63.

Question 2
Calculate the sequence from a 5 bit LFSR using feedback taps 5 & 3.

This is the same as for SG UP:UP sequence

[1111100011011101010000100101100]

Question 3
(a) For the set up in Fig11, write down an expression for x(t) in terms of the input y(t) and the S.U.I. impulse response h(t).

Convolution y*h

(b) Write down an expression for the CCF of x and y, and substitute the expression for x from (a).

(c) Demonstrate that the result in (b) can be reduced to the convolution of h(t) and the ACF of the input.

(d) Show that if the ACF of y is an impulse function, the output of the cross-correlator gives h(t) (with a scaling factor).

(e) Demonstrate that if the input is white noise the ACF is an impulse

Question 4
(a) In the term "matched filter" what are the items that are matched?

the impulse response of the MF is matched to the pulseform of the data symbol at its input

(b) What is the role of the MF in a digital communication receiver?

Pre-lab preparation work

The MF provides the best SNR at the decision instant, hence the least

probability of error.

(c) describe the operation of the "integrate & dump" process in a digital communication receiver

(d) explain why the I&D receiver is effectively a filter with a square pulse as its impulse response

(e) extend (d) to explain why the I&D receiver is the matched filter for square pulseform data sequences in additive white noise

Question 5

What voltage is the output of the MULTIPLIER ? Explain why this is so

4V.

+2V * +2V = 4V, or -2 * -2 = 4V

Question 6

What voltage would the ramp have reached if it had not saturated ?

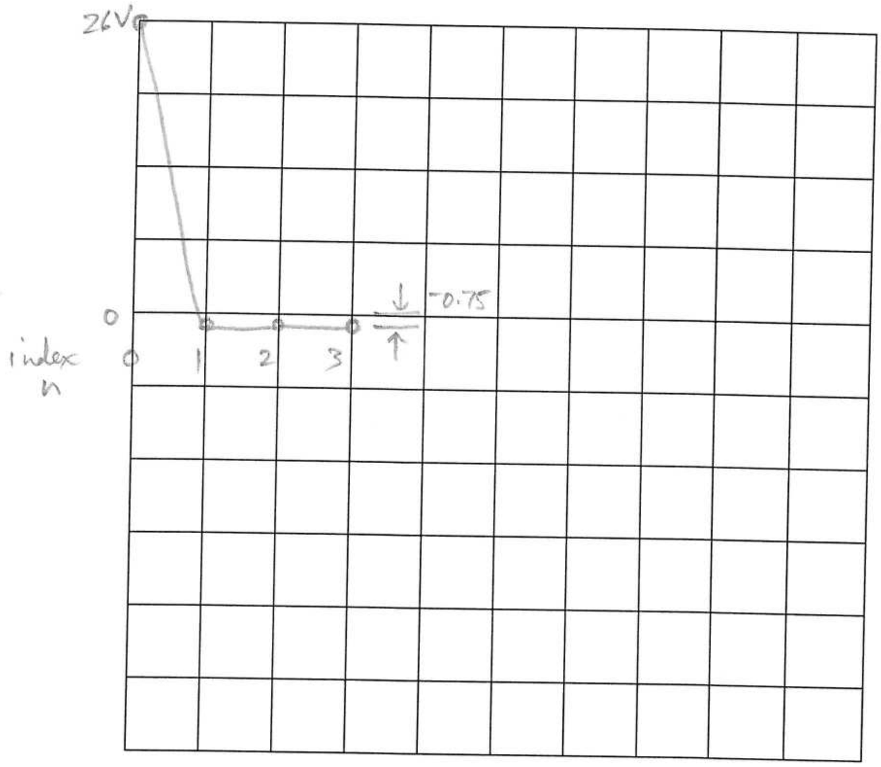
Rate = 10.5/4ms = 2625 V/s, so for a period of 10ms

2625 * 10ms = 2625 * 0.01 = 26.25V

Question 7

What voltage is at the I & H output ?

-0.75 V



Experiment X 6 : Graph 1

Graph 1: Plot of I & H voltages vs. delay position n

Question 8
How well do these results correspond with your theoretical expectations from the pre-lab preparation work ?

Very well. Theory expects 31:-1, measurements give 26:-0.75. If we scale these by 31/26

it gives 31:-0.9

Question 9
How well do these results correspond with your theoretical expectations ?

Similar relationships.

Question 10
Based on your measured ACF, what can you say about this sequence ?

26V;4.1V; -2.5V; -2.5V. This sequence is not maximal length

It probably includes repetition.Will not have a uniform spectrum.

Question 11
What observations can you make about this signal from its ACF ?

Alignment occurs at n=18.ACF as for a maximal length sequence.

Question 12
What observations can you make from this cross-correlation about the nature of the two sequences ?

Not correlated at all. Very different sequences.

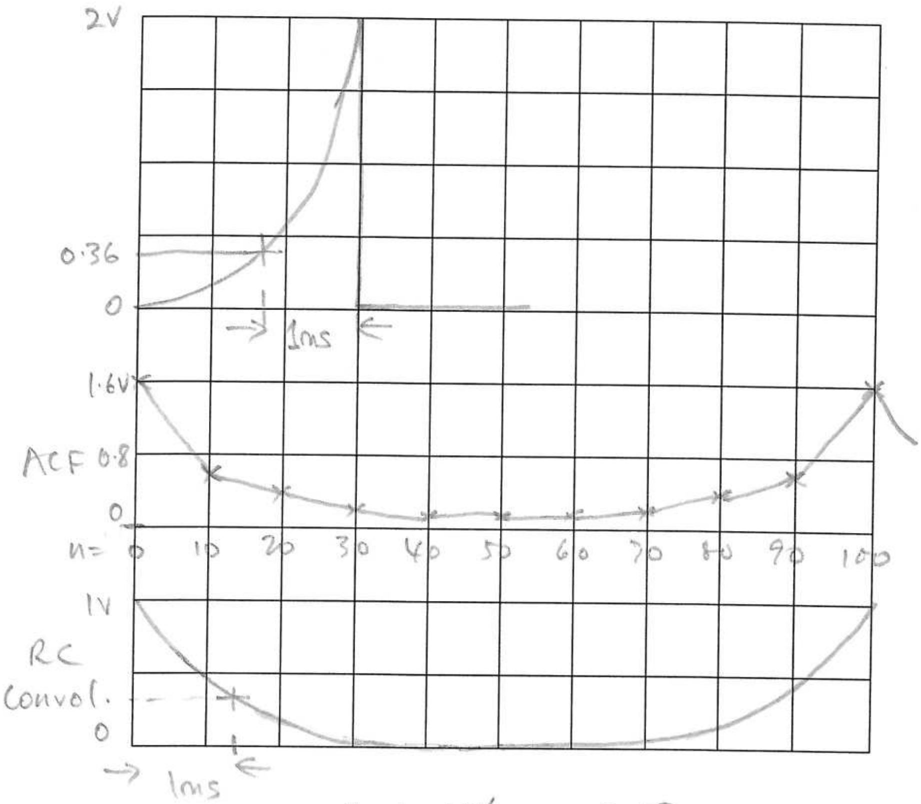
Question 13
Write down the 31-bit pattern for both PRBS sequences here. Note also the number of bit pattern "runs" ? Why is the pattern "00000" not present ?

SG-PRBS:[1111100011011101010000100101100]

ALT-PRBS: [1111100110100100001010111011000]

4 runs of 0 & 1; 2 runs of 00 & 11; 1 run of 000 & 111; 0 runs of 1111; 1 run of 0000;

1 run of 11111; 0 runs of 0000 as 0000 is illegal. 0000 would cause the LFSR to stop.



Experiment X 6 : Graph 2

Graph 2: using exponential pulses

Question 14

How could you describe this function ?

A "delta function", as it is prominent only at one point

Question 15

Is the bandwidth of the proposed input for this exercise adequate for this application ?

Pulse rate = 3.3kHz.Time constant of RC NETWORK impulse response is around 1ms, so 3dB

Bandwidth of SUI (RC) = 1000rad/sec = 160Hz. Hence 3300/160 = approx.20 = adequate.

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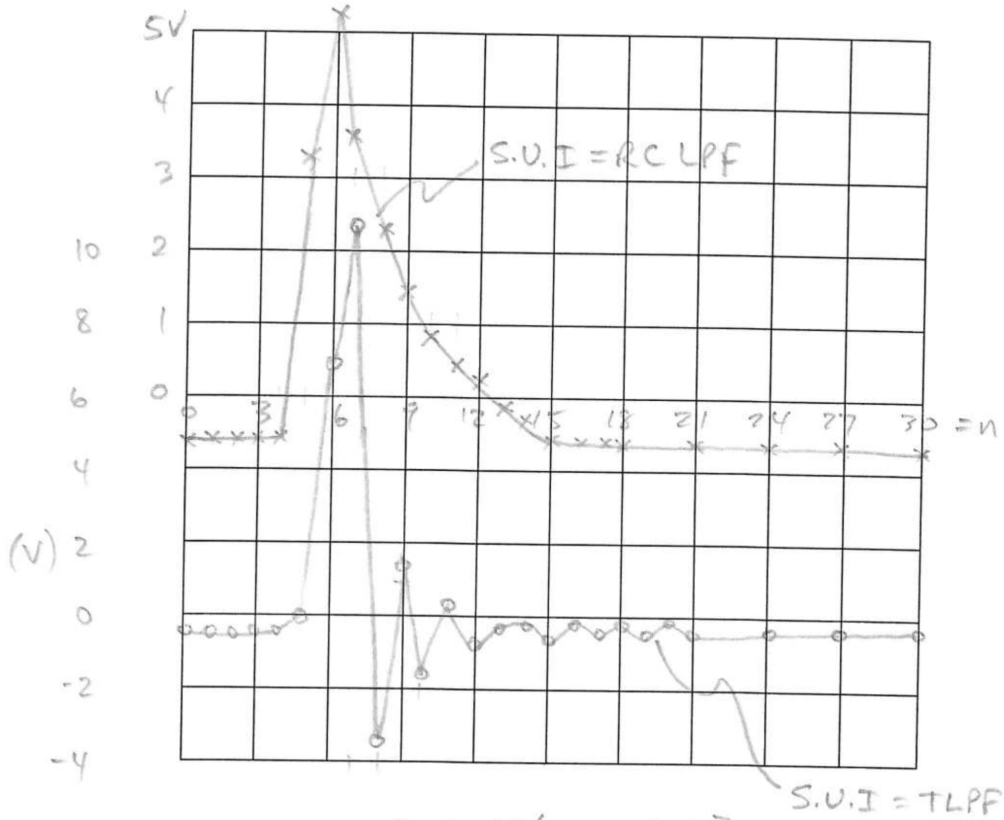


Experiment X 6 : Graph 3

Graph 3: in/out correlation plots

Question 16

Describe the output waveform from the correlator for the RC NETWORK SUI ?

Resembles the impulse response of an RC NETWORK.

Question 17
Describe the output waveform from the correlator for the TUNEABLE LPF ?

Resembles the impulse response of the TUNEABLE LPF, including the ringing.

Question 18
How many errors do you estimate are occurring in the recovered data signal after the filter ?

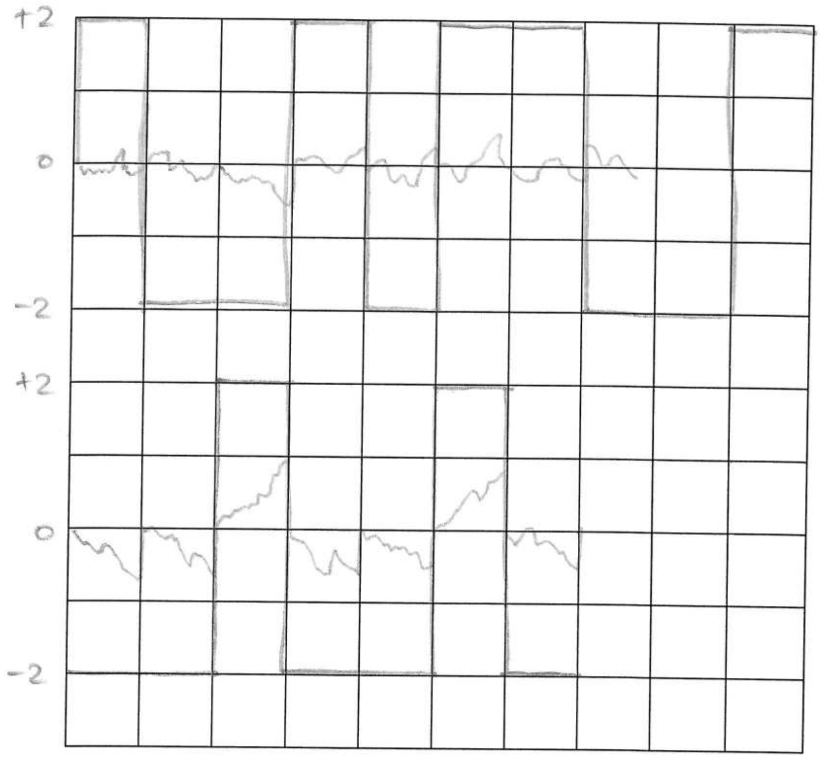
Zero. The final point of the integration is well clear of the decision boundary at 0V.

Statistically, there may be errors however we are not covering that issue here.

Question 19
How do you determine when errors are occurring ? At what signal levels did this occur ?

Visually, ifthe end point ofthe integration, at the decision instant only, has crossed over the 0V threshold.Temporary excursions do not cause errors eg near 3 ms in Figure 17

When signal gain = 0.1, and noise gain = 2 (max), it is possible to see occasional crossovers.



Experiment X 6 : Graph 4

Graph 4: integrate & dump filtering

References

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X7 - Exploring complex numbers and exponenetials

SIGEX

Pre-lab preparation

Question 1

Confirm your understanding of the algebra associated with complex number by solving these equations using the binomial method :

a) (3 + i2) + (5 – i6)   b) (3 + i2) x (5 – i6)   c) (3 + i2) - (5 – i6)
d) (a + ib) + (c + id)   e) (a + ib) x (c + id)   f) (a + ib) x (a – ib)

8 – i4; 27 – i8; -2 +i8;

(a+c) + i(b+d); (ac-bd) + i(bc + ad); $a^2 + b^2$

Question 2

Confirm your understanding of the algebra associated with complex number by solving these equations using vectors. Sketch your working on the graph below:

a) (3 + i2) + (5 – i6)   b) (3 + i2) x (5 – i6)   c) (3 + i2) - (5 – i6)
d) (a + ib) + (c + id)   e) (a + ib) x (c + id)   f) (a + ib) x (a – ib)

a) (3 + i2) == 3.6//33degrees

f) (a+ib)x(a-ib)=conjugates= $a^2 + b^2$//0 degree

Question 3

Write the equation for signals at DAC-1 and DAC-0 as a function of time in the form: A.cos(wt + θ). Think of the centre of the scope timeline as the instant t=0.

DAC-1 = 1.cos(wt+0)

DAC-0 = 1.cos(wt-(2π.90/360)); 2π.90/360 is 90 degrees expressed in radians

Question 4

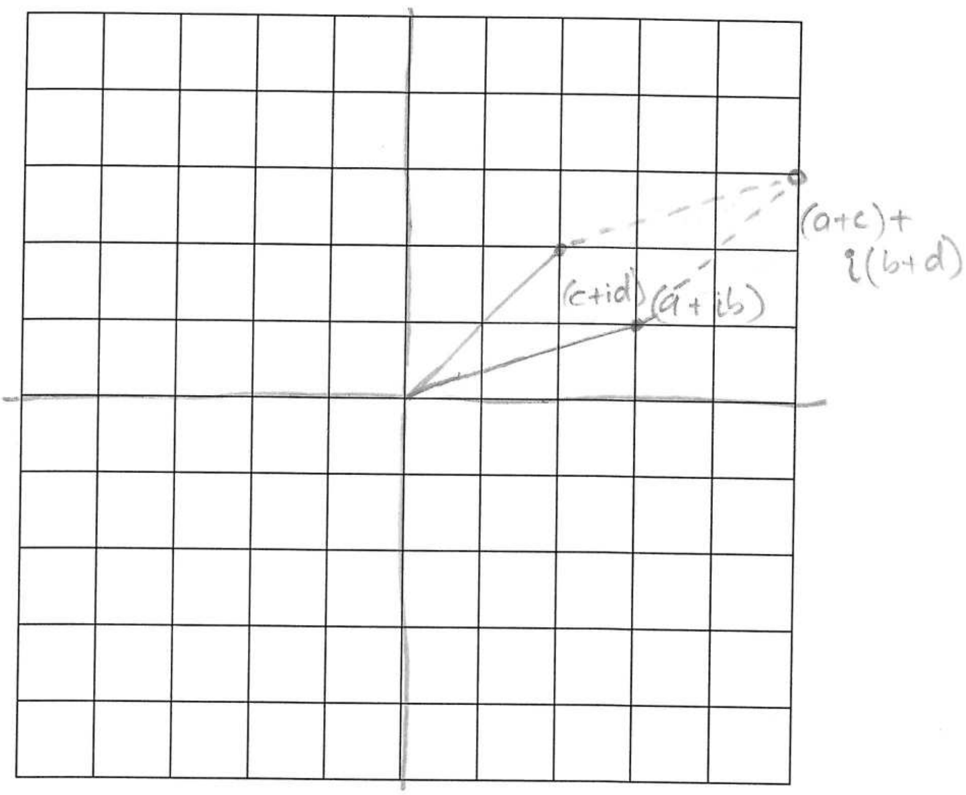
Explain why the XY graph displays a circle ?

Locus of sinwt vs. coswt inscribes a circle.

Question 5

Explain the signal as viewed on the XY graph ?

DAC-1 drives the horizontal X axis of the X-Y display, whilst DAC-0 drives the vertical Y axis.



Experiment X 7 **: Graph** 1

Graph 1: Vector arithmetic

Question 6

Write the equation for signals at DAC-1 and DAC-0 as a function of time in the form: A.cos(wt + θ).

DAC-1 = 1.cos(wt-15deg)

DAC-0 = 1.2cos(wt+75deg); Degrees shown for convenience.Should be expressed in radians.

Question 7

Measure and document the equation for the sum of the two sinusoids. Compare this with the expected resultant using the phasor method.

Define DAC-1=1cos(wt+0), then f+g = 1.4.cos(wt+45deg), as phase difference is

1.25/10*360 = 45 degrees or π/4 radians. Phasor gives the same.

Question 8

What is the output sum signal for these settings ? Is this expected ? Explain.

0. expected as the signals null each other.

+180 degree shift is the same as -180 degree shift.

Phase (degrees)	Resultant amplitude (Vpk)	Phase (degrees)	Resultant amplitude (Vpk)
0	2	210	-1.75
30	1.75	240	-1
60	1	270	0
90	0	300	1
120	-1	330	1.75
150	-1.75	360	2
180	-2		

Table 1: resultant amplitude readings

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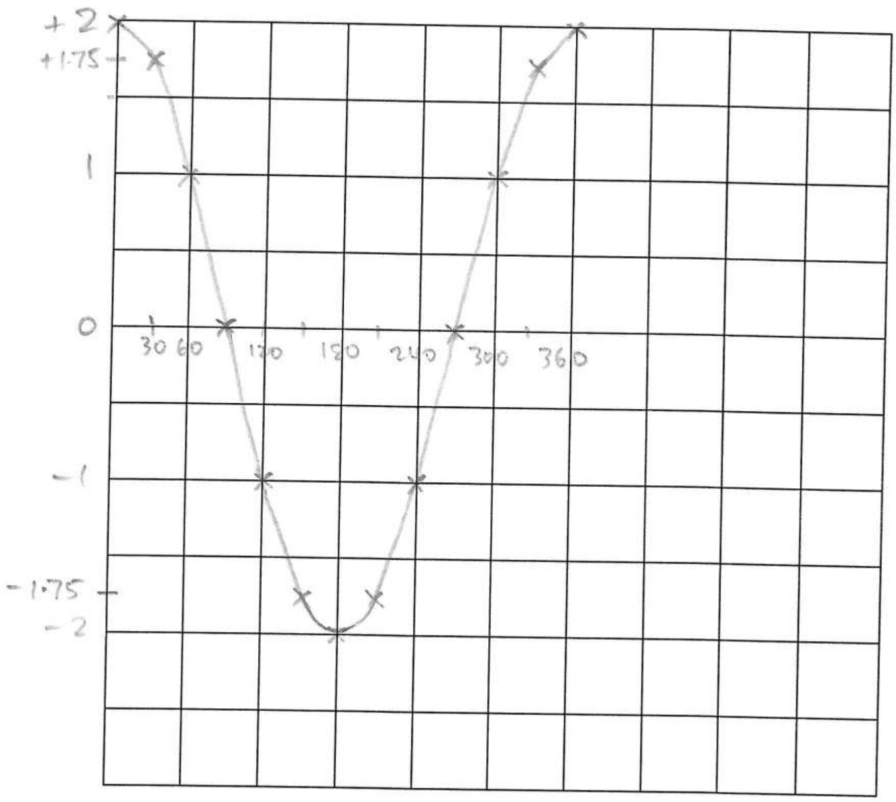


Experiment X 7 : Graph 2

Graph 2:plot of resultant from measurements

Question 9

What is the equation for this resultant signal ?

2.cos(wt)

Question 10

What is the equation for this resultant signal for a1=0.5 ?

$N(t) = 4.5 .2^{-1000t}$

Question 11

What is another term for the time constant when a1=0.5 ?

Halflife

Name: ______________________________

Class: ______________________________

X8 - A fourier Series analyser

SIGEX

Experiment 8 – A Fourier Series analyser

Question 1
How would you expect the summation of to look if you could add up many more harmonics ?

Similar shape, with maximum reaching "n" for n harmonics.

Corners becoming squarer.

Question 2
What is its peak amplitude and is this as expected ?

10V. Yes, as cosine harmonics are equal to 1 at t=0.

Question 3
Is the fundamental an odd or even function ? Is the summation odd or even ?

Even. Even

Question 4
Write the equation for the summation of the 10 signals ? Is it symmetrical about the X axis?

cos(1wt) + cos(2wt) + cos(3wt) + …+cos(10wt)

No.

Question 5
Vary the amplitudes and notice how the signal changes . You may set the amplitude of certain components to 0 as you see fit. Can you create a wave form which starts at a zero value ? Write the equation for your new varied amplitude signal ? Does it start at a zero value ?

Will never start at a zero value.

Question 6
How would you expect the sine summation of to look if you could add up many more harmonics ?

Mainly zero level with positive and negative impulses at the fundamental.

Question 7

What is its peak amplitude and is this as expected ? Is this an odd or even function ?

Approx. 7.5V.

Odd

Question 8
Vary the amplitudes and notice how the signal changes . You may set the amplitude of certain components to 0 as you see fit. Can you create a wave form which starts at a non-zero value ? Write the equation for your new varied-amplitude signal ? Does it start at a non-zero value ? Is it symmetrical about X axis.?

No.All sinewaves harmonics are equal to 0 at t=0.

Yes

Question 9
Write the equation for the summation of these 2 waves ? Write the equation for the summation in terms of the sine wave with a non zero phase shift.

Sin(wt) + cos(wt) = 1.4sin(wt + 45degrees) = 1.4sin(wt + π/4)

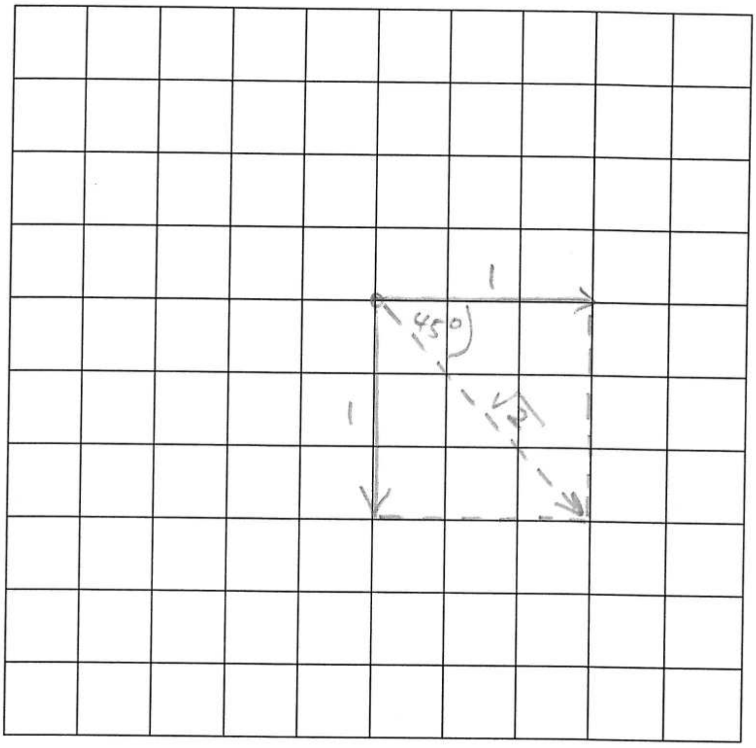
Question 10
Describe how the summation changes as you vary the respective amplitudes?

The resultant amplitude & phase vary.

Question 11
For a particular pair of amplitudes you have set, write the equation for the summation in terms of sine and cosine as well as its equivalent polar representation ?

Sin(wt) + cos(wt) = 1.4sin(wt + 45degrees) = 1.4sin(wt + π/4)

1//-90deg + 1//0deg = 1.4//-45deg



Experiment X 8 **: Graph** 2

Graph 2: components & resultant

Question 12
What is the output value at the end of the integration period ? HINT: the I&H function will hold the final value.

Zero volts.

Question 13
What is the average value of these three products ?

Question 14
What is the average value of these products ?

Non-zero, approx 2 V, however note that the TLPF gain has not been set to unity, so the only

Valid conclusion is that the result is non-zero.

Question 15
Write the complete formula for the product of a cosine, Acoswt, by itself? What do the terms represent?

$\cos(wt) * \cos(wt) = \frac{1}{2}\cos(2wt) + 1/2$

Cos(2wt) is the sum term with 0 average, and ½ the difference term, which is also a DC component.

Harmonic number	sine (V)	cosine (V)
1st	0	1
2nd	0.3	0
3rd	1	0.5
4th	0	0
5th	0	0
6th	0	1
7th	2	0
8th	0	0
9th	0	0
10th	0	0
DC (V) =	n/a	n/a

Table of measured coefficients

Question 16
How do your readings compare with expectations ? . Explain any discrepancies .

Very accurate.

Question 17
What do you notice about their phase relationship ? Is this to be expected ? Explain.

They are drifting relative to each other. As they are not synchronized, this is to be expected.

Input frequency (Hz)	TLPF output pp swing (V)	Half of pp (V)	Entered values	Calculated resultant (V)
100	2	1	1;0	1
200	0.6	0.3	0;0.3	0.3
300	2.2	1.1	0.5;1	1.11
400	0	0	0;0	0
500	0	0	0;0	0
600	2	1	1;0	1
700	4	2	0;2	2
DC (V) =	-	0.45	0.5	0.5

Table of measured coefficients

Question 18

Can you explain if your readings differ in some places from the actual value ?
HINT: you have one value per harmonic instead of two. Consider the previous discussion above about resultants in your answer. And to allow for MULTIPLIER and TLPF gains.

Readings are accurate. Measurement error may arise.

Value at 300Hz is a resultant of 2 orthogonal components.

Input frequency (Hz)	TLPF output amplitude (V)	Scaled measured values (V)	Calculated resultant (V)
100	1.6	1	1
200	< 0.1	-	0
300	0.5	0.31	1/3 = 0.33
400	< 0.1	-	0
500	0.3	0.19	1/5 = 0.2
600	< 0.1	-	0
700	0.2	0.125	1/7 = 0.14
DC (V) =	2.25	2.25	2.4

Table of measured coefficients for squarewave

Question 19
Why are some of the harmonics hard to detect ?

They are quite small, and even harmonics are not present.

Question 20
Can you now detect even harmonics in the squarewave of 20% duty cycle ?

Yes, they are now present.

Question 21
Compare your measured coefficients for the first 4 odd harmonics as ratios to that expected by theory ? Remember to normalize the measurements for the comparison.

Comparison in the table above. Results are close to theory after normalizing.

References

Langton.C.,"*Fourier analysis made easy*", www.complextoreal.com

Name: ______________________________________

Class: ______________________________________

X9 - Spectrum analysis of various signal types

SIGEX

Experiment 9 – Spectrum analysis of various signal types

Pre-requisite work

Question 1
What is the conversion equation for a linear voltage scale to a logarithmic scale ?

Log(dB) = $20\log_{10}(V2/V1)$; where V2/V1 is a linear ratio

Question 2
What linear ratio does a -6dB gain equal ?

-6dB = $20\log_{10}(V2/V1)$; so V2/V1 = 10exp(-6/20) = 0.5

Saying a level has reduced by -6dB is equivalent to saying it has halved. (+6dB == doubling)

Question 3
List some of the more important characteristics of PN sequences:

Maximal length sequences with n stages (LFSR) repeat every 2^{n-1} clocks. Maximal length

Sequences are orthogonal ie: are correlated at only one point. # runs is well defined.

Question 4
Multiply a sinewave with a squarewave so as to create a halfwave rectified sinewave and calculate its spectrum:

Question 5
At what frequencies do the nulls occur at ?

n x 5000; n=1,2,3.....

Question 6
What is the mathematical relationship between null spacing and the pulse width ?

n x 1/pulse width ; n=1,2,3,...

Question 7
What are the characteristics of the sin(x)/x form that you are looking for ?

Pre-lab preparation work

Zero crossing points (sin x), amplitude of envelope (1/x)

Question 8
What is the general trend that you are observing as the duty cycle tends towards 0 ?

Null being pushed out to infinity, and spectrum envelope tending to become a constant level.

Question 9
Using the various findings so far, what shape you expect the spectrum of a single pulse, that is, a pulse train with very large separation between pulses, to have ?

It should have components to infinity, all with a constant amplitude.

Question 10
At what time instants does the sync pulse have a zero crossings ?

Every 0.1 ms

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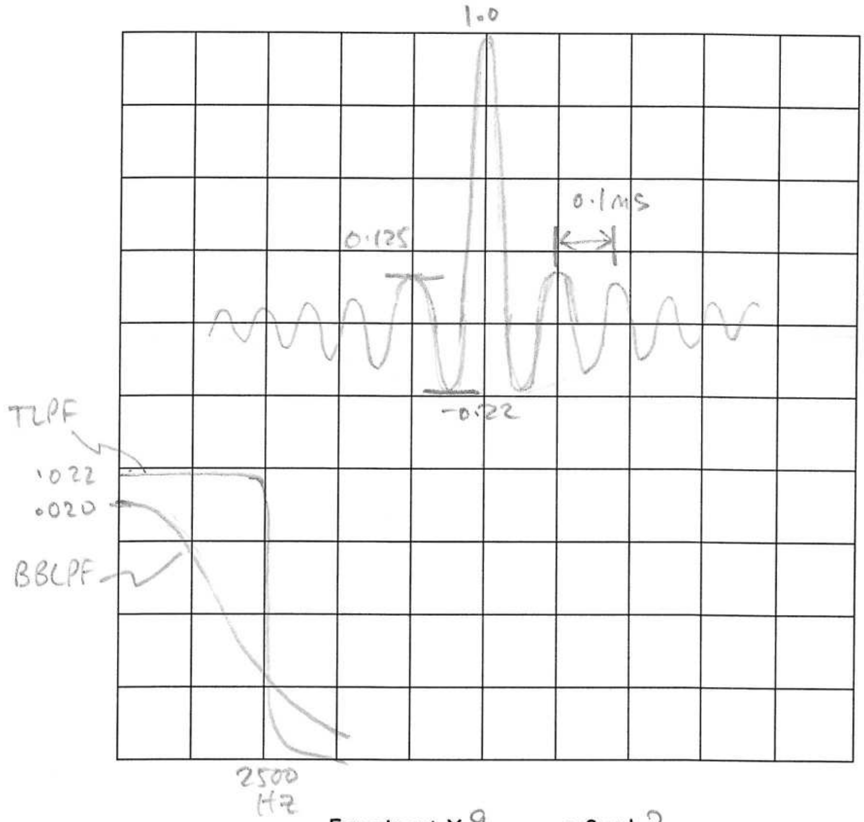

Experiment X 9 : Graph 2

Graph 1: sync pulse train time and frequency responses

Question 11

Why do you suppose state [00000] is illegal ?

The system would lock and never change state from 00000

Question 12
Where do the nulls occur ? What is the separation between harmonics ? What do these values relate to ?

At multiples of the clock rate, 2000, 4000, 6000 Hz, …

Measured to be approx 66Hz.Theory states 2000/31 = 64.5 Hz

Question 13
Where do the nulls occur ? What is the separation between harmonics ? What do these values relate to ?

At multiples of the clock rate. These cant be measured.The separation should be

Clk rate/16383 Hz…too close to measure with our current setup.

Question 14
How many harmonics are visible in the filter output ?

5 or 6. Not satisfactory, too repetitive. Period too short.

Question 15
How many harmonics are visible in the filter output ? Calculate this.

Seperation =2000/16383 = 0.122Hz. Impossible to determine visually.

250Hz/0.12 = 2047 harmonics.

Question 16
Is this analog noise signal periodic ? What is its period ? Calculate this

Yes, though it is not measurable with current setup.

Period = 16383 x 1/2000 = 8.2 seconds. (NB 8.2 sec = 1/0.122 from previous question)

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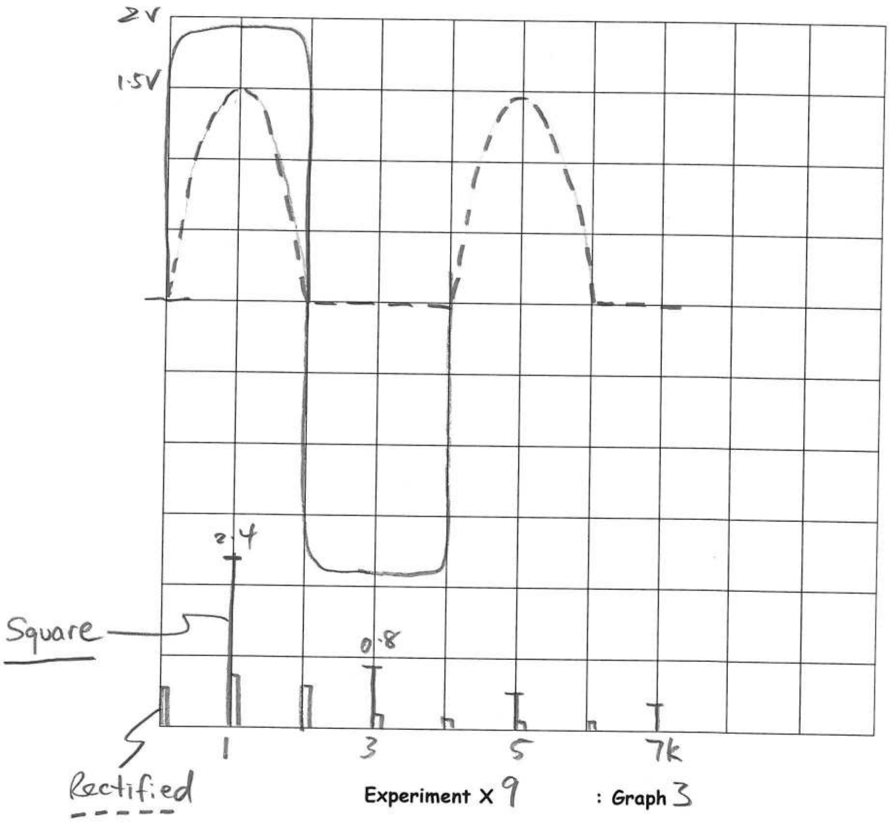


Graph 2: clipped signals and spectra

Question 17
What effect does a higher level of clipping have on the spectrum of the clipped signal ?

Harder clipping gives more spectral harmonics

Question 18
What is the relationship between the input frequency and the output harmonic frequencies ?

Harmonics occur at integer multiples of the input frequency.

Question 19
What can you say about the spectrum of the rectified sine wave ? Is this what you would have expected ? Refer back to your pre-lab preparation questions.

Yes. As expected. Far less harmonics than for the clipped case.

Question 20
Is the clipping process a linear or non-linear process ? Explain.

Non-linear. Clipping creates harmonics which were not existent in the original signal.

Name: ______________________________________

Class: ______________________________________

X10 - Time domain analysis of an RC circuit

Experiment 10 – Time domain analysis of an RC circuit

Pre-requisite work

Question 1: the step response
This question follows the integration method in Section 4 of S.K.Tewksbury's notes
http://stewks.ece.stevens-tech.edu/E245L-F07/coursenotes.dir/firstorder/cap-difeq.pdf

(a) Apply elementary circuit theory to show that the voltage equation for the RC circuit in Figxxx is

$V_{in}(t) = i(t).R + V_{cap}(t) = i(t).R + Q(t)/C$ Eq prep1.1

Where $Q(t)$ is the charge in capacitor C.

(b) Show that this can be expressed as

$(d/dt)(V_{in}) = R.di/dt + i/C$

Consider the case where $V_{in}(t)$ is a step function of amplitude V_o and the capacitor charge $Q(t) = 0$ at $t = 0$. Show that for $t > 0$ $(d/dt)(V_{in}) = 0$ and the DE reduces to

$di/dt = - a.i$ $[a = (1/RC)]$

Use $(d/dt)\log_e(i) = 1/i$ to show that the solution of the DE is

$i(t) = i_o \exp(- a.t)$ $(t > 0)$ $[i_o = V_o/R]$

(c) Use Eq 1.1 to show that
$V_{out}(t) = V_{cap}(t) = V_{in}(t) - R.i_o \exp(- a.t)$

Hence the step response $V_{out}/V_{in} = (1 - \exp(- a.t))$

(d) Plot the result in (c) for $a = 1000$

(e) What is the asymptotic value of the step response as t increases indefinitely? Show that the step response rises to $(1 -1/e)$ of its final value at $t = 1/a$.

SIGEx Expt 10 RC in Time Domain

Prep solutions P1

P1 (a) and (b)

For V_{in} = step function $\frac{d}{dt} V_{in} = 0$ for $t > 0$

$\Rightarrow \frac{di}{dt} = -\alpha i \Rightarrow \frac{di}{i} = -\alpha\, dt$

$\Rightarrow d(\log_e(i)) = -d(\alpha t) \Rightarrow \log_e(i) = -\alpha t + \text{constant}$

$\Rightarrow i = \exp(-\alpha t + \text{constant}) = e^{\text{const.}} e^{-\alpha t}$

$= i_o e^{-\alpha t}$

At $t = 0$ $i = i_o = \frac{V_o}{R}$ (since $V_{cap} = 0$)

(V_o is the amplitude of the input step function)

Confirm that $i = i_o e^{-\alpha t}$ is a solution of the DE:

$LHS = \frac{di}{dt} = \frac{d}{dt}(i_o e^{-\alpha t}) = -\alpha i_o e^{-\alpha t} = -\alpha i = RHS$

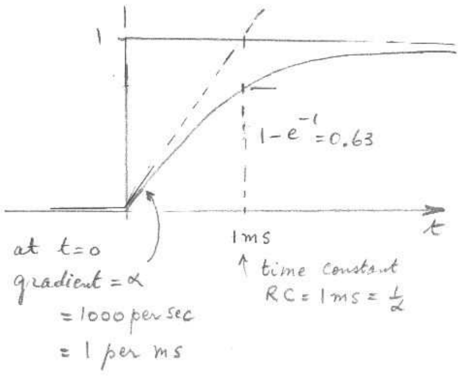
P1(c)

$V_{out}(t) = V_{in}(t) - R i_o e^{-\alpha t}$

For $t > 0$ $V_{in} = V_o \Rightarrow \frac{V_{out}}{V_{in}} = 1 - \frac{R i_o}{V_{in}} e^{-\alpha t}$

$\Rightarrow \frac{V_{out}}{V_{in}} = 1 - \frac{R i_o}{V_o} e^{-\alpha t} = 1 - e^{-\alpha t}$ since $i_o = \frac{V_o}{R}$

P1(d) – (e)



at $t = \frac{1}{\alpha}$ response $= 1 - e^{-1} = 0.63$

$t \to \infty$ response $\to 1$

$t = 0$ $\frac{d}{dt}(\text{response}) = \alpha$

$= 1000/\text{sec}$

$= 1/\text{ms}$

[i.e. gradient line projection = 1 at t = time constant = $\frac{1}{\alpha}$]

END P1

Question 2: the impulse response

(a) Describe the main properties of the theoretical impulse function.

Show that differentiation of the unit step function wrt t produces a unit impulse at t = 0. Apply this to the step response result in Question P1(c) to show that the impulse response h(t) of the RC circuit is

a. exp(- a.t)

SIGEx Expt 10 RC in the Time Domain

PREP SOLUTION P2 (impulse response).

(a) Main properties of the theoretical impulse function.

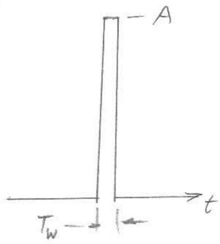
(i) $\delta(t) = 0$ for all t except $t = 0$
At $t = 0$ $\delta(t) = \infty$

(ii) $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

(iii) convolution $\int_{-\infty}^{t} \delta(\tau)\, x(t-\tau)\, d\tau = x(t)$

It is evident that the derivative $\frac{du}{dt}$ of the unit step function behaves like an impulse in respect of the above properties.

$\frac{d}{dt}(\text{step resp}) = \frac{d}{dt}\left(1 - e^{-\alpha t}\right) = -\left(-\alpha e^{-\alpha t}\right) = \alpha e^{-\alpha t}$



(b) It is not possible to produce a pulse with infinite amplitude and zero width. A rectangular pulse provides a practical approximation. To be useful for measuring impulse responses the pulse width T_W must be at least an order of magnitude smaller than the duration of the smallest feature of the response. However, if T_W is excessively small, the response amplitude may be inadequate for displaying on a scope.

For the RC circuit in Q. P1, T_W should be $\lesssim \frac{(1/RC)}{10}$ i.e. not greater than around 100 μsec.

We can think of the finite pulse width T_W as causing a blurring of the response, e.g. as with an out of focus camera.

(b) Explain why the impulse function can only be approximated in practice.

Sketch an impulse approximation realized as a finite width pulse. Explain why an excessively narrow pulse is undesirable in practical applications. Estimate a pulse width that would be suitable for use with the case in Question P1. Indicate your reasoning.

(c) Using the property in (a) we could generate the impulse response by first recording the step response, then differentiating. Compare this alternative with the use of a finite width pulse input. Include discussion of signal peak limitations and output amplitude considerations.

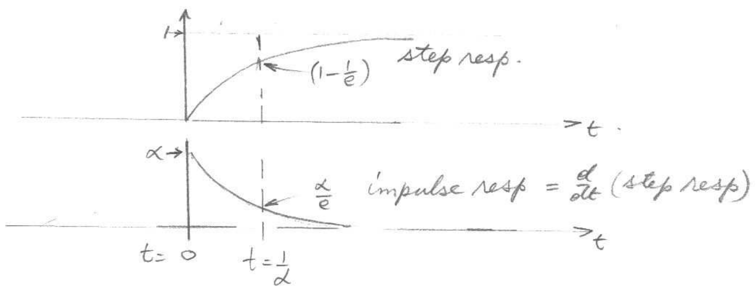
(d) Show that the impulse response falls to 1/e of its initial value at
t = 1/a

SIGEx Expt 10 RC in the Time Domain

Prep Solution P2 (page 2/2)

(c) Generating a step response is straightforward, likewise differentiation (e.g. using a digital scope). A major advantage of this approach is that large input peaks are avoided. The switching time of the step needs to be adequately fast.

(d)



From P1

Step response $= 1 - e^{-\alpha t}$

$\frac{d}{dt}$(step resp) $= \alpha e^{-\alpha t} = h(t)$

at $t = \frac{1}{\alpha}$ $h(t) = \alpha / e = \frac{1}{e} h(0)$

END P2.

Question 3: convolution and response to an exponential input

This question introduces convolution and its application in the analysis of systems like the RC circuit in Q. P1.

(a) The convolution of the time functions x_1 and x_2 can be expressed as

$$x\_1 * x\_2 = \int_0^t x\_1(\tau) \,.\, x\_2(\tau - t)\, d\tau \qquad \text{[for } t > 0\text{]}$$

Note that the convolution is a function of t and that tau is a dummy variable that has no further role after integration.

Show that changing the order (x_2 * x_1) does not change the result.

Show that if x_1 is a unit impulse the convolution x_1 * x_2 = x_2(t).

Suppose we approximate a continuous time signal $x_1(t)$ as a sum of very narrow contiguous pulses, each of which can be thought of as representing an impulse function (each with its individual amplitude). Suppose next that this pulse train representation of $x_1(t)$ is then applied as input to the system introduced in Q. P1. Each of the pulses in the train will produce an individual output that will be a close (weighted) approximation to the system's impulse response. The overall output will be the sum of these (overlapping) weighted impulse response approximations.
Demonstrate that this sum is effectively the convolution of x_1 and the system's impulse response $h(t)$. (Invoke the usual limit methods to morph the discrete sum into a continuous time integral.)

(b) Show that for $t > 0$, the convolution for the case

$x_1(t) = \exp(- a1.t)$ and $x_2(t) = \exp(- a2.t)$ [a1 N.E. a2]

is $(1/(a2 - a1)) . (\exp(- a1.t) - \exp(- a2.t)$

(c) Sketch the graph of the result in (b) versus t over the range $t > 0$. Show that for positive values of a1 and a2 the function is positive for $t > 0$, and that it is zero at $t = 0$ and $t \to$ infinity. Find the peak and the corresponding value of t for $a1 = 0.5$ and $a2 = 1.1$.

(d) Use the results in (a) and (b) and in Q. P2(a) to obtain the response of the RC circuit in Q.P1 when the input is

$$x_1(t) = \exp(- a1.t)$$

(e) Repeat the tasks in (b) and (c) for the case $a1 = a2 = a$.

NB: a useful reference for this question is Schuam Laplace Transforms (1965); p45 (convolution of two exponentials)

SIGEx Expt 10 RC in Time Domain

Prep Solutions P3(a)-(b)

P3(a) Changing the order does not affect the result,

i.e., $x_2 * x_1 = x_1 * x_2$.

Proof:

$$x_2 * x_1 = \int_0^t x_2(\tau)\, x_1(t-\tau)\, d\tau$$

Put $\tau' = t-\tau \Rightarrow x_2 * x_1 = \int_{\tau=0}^{\tau=t} x_2(t-\tau')\, x_1(\tau')\, d(t-\tau')$

$$= -\int_{\tau'=t}^{\tau'=0} x_1(\tau')\, x_2(t-\tau')\, d\tau' .$$

$$= \int_0^t x_1(\tau')\, x_2(t-\tau')\, d\tau' = x_1 * x_2 \quad \text{QED}.$$

Convolution with an impulse function.

$$x_1 * x_2 = \int_0^t \delta(\tau)\, x_2(t-\tau)\, d\tau \qquad (t \geq 0)$$

$$= \int_0^t \delta(\tau)\, x_2(t)\, d\tau \quad \left[\begin{array}{l}\text{integrand} = 0\\ \text{except at } \tau = 0\end{array}\right]$$

$$= x_2(t) \int_0^t \delta(\tau)\, d\tau$$

$$= x_2(t) \quad \text{since} \int_0^t \delta(\tau)\, d\tau = 1$$

P3(b) Convolution of two exponential functions

$x_1 = e^{-\alpha_1 t}$ $\qquad x_2 = e^{-\alpha_2 t}$

$$x_1 * x_2 = \int_0^t e^{-\alpha_1\tau} e^{-\alpha_2(t-\tau)}\, d\tau = e^{-\alpha_2 t} \int_0^t e^{(\alpha_2-\alpha_1)\tau}\, d\tau$$

$$= \frac{e^{-\alpha_2 t}}{\alpha_2-\alpha_1}\Big[e^{(\alpha_2-\alpha_1)\tau}\Big]_0^t$$

$$= \frac{1}{\alpha_2-\alpha_1}\left(e^{-\alpha_1 t} - e^{-\alpha_2 t}\right) \qquad t > 0,\ \alpha_2 \neq \alpha_1$$

SIGEx Expt 10 RC in Time Domain

Prep Solutions P3(c)

P 3(c) – (e)

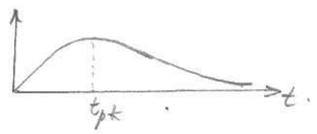
$\alpha_2 > \alpha_1$: den > 0 and $e^{-\alpha_1 t} > e^{-\alpha_2 t} \Rightarrow x_1 * x_2 > 0$

$\alpha_2 < \alpha_1$: den < 0, $e^{-\alpha_1 t} < e^{-\alpha_2 t} \Rightarrow x_1 * x_2 > 0$

$[\ \alpha_1 > 0 \quad \alpha_2 > 0 \quad t > 0\ ]$.

By substitution the result is zero at $t = 0$ and $t \to \infty$

Hence there will be a maximum.



We can find the maximum by differentiation.

$$\frac{d}{dt}\, x_1 * x_2 = \frac{1}{\alpha_2 - \alpha_1}\left[(-\alpha_1)e^{-\alpha_1 t} - (-\alpha_2)e^{-\alpha_2 t}\right] = 0$$

$$\Rightarrow \alpha_1 e^{-\alpha_1 t} = \alpha_2 e^{-\alpha_2 t} \Rightarrow -(\alpha_2 - \alpha_1)t = \log_e\left(\frac{\alpha_2}{\alpha_1}\right)$$

$$\Rightarrow \text{maximum at} \quad t_{pk} = \frac{1}{\alpha_1 - \alpha_2}\log_e\left(\frac{\alpha_2}{\alpha_1}\right)$$

For $\alpha_1 = 0.5$, $\alpha_2 = 1.1$ $\quad t_{pk} = 1.3141$

$$\text{Gradient } (t = 0) = \frac{\alpha_2 - \alpha_1}{\alpha_2 - \alpha_1} = 1$$

P 3(d) Let the time constant of the RC circuit $= \frac{1}{\alpha_2}$

Output $V_{out} = x_1 * h$ where $h(t) = V_0 e^{-\alpha_2 t}$ (impulse response)

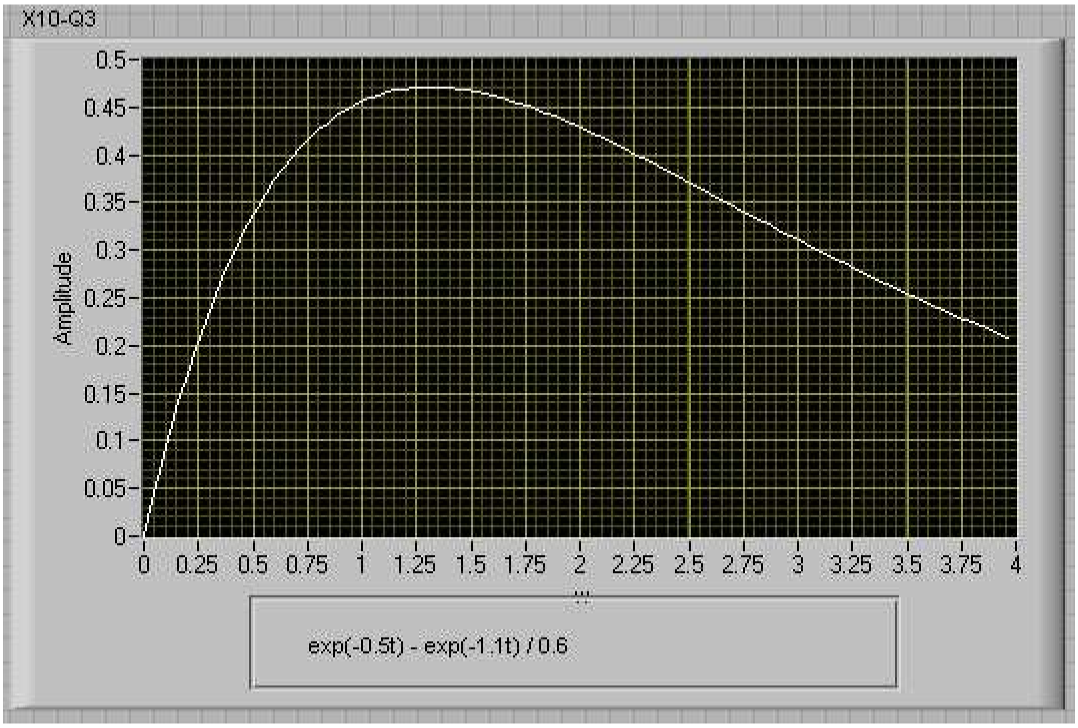
$$\Rightarrow V_{out} = \int_0^t e^{-\alpha_1 t} \cdot V_0 e^{-\alpha_2 (t-\tau)}\, d\tau$$

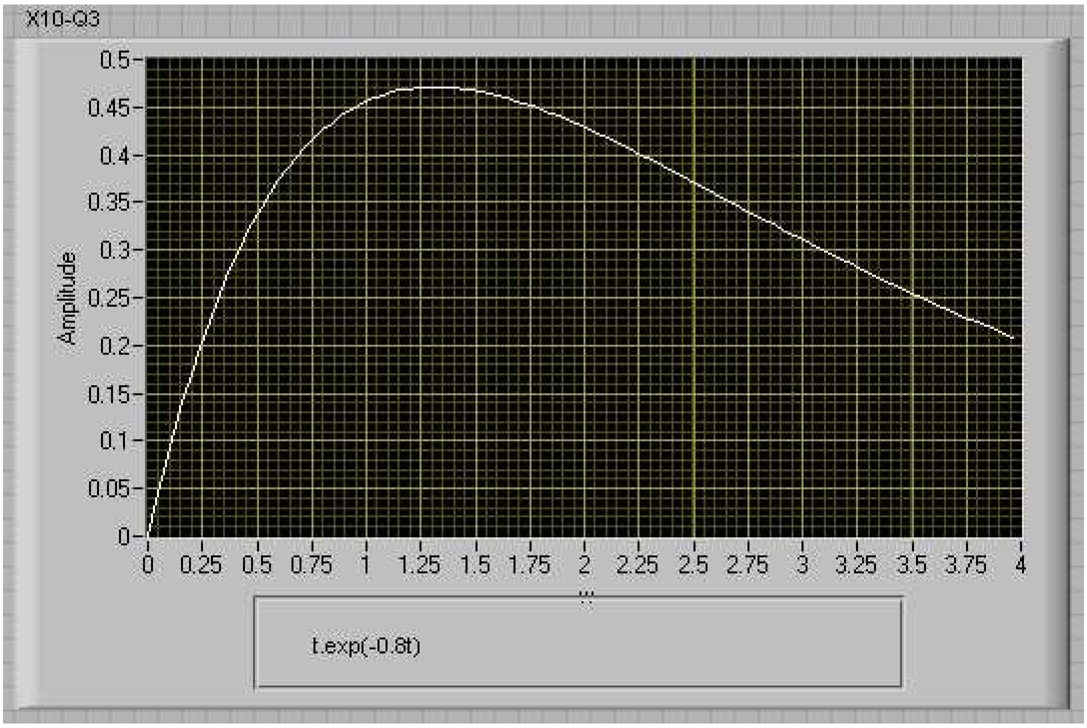
$$= V_0 \cdot \frac{1}{\alpha_2 - \alpha_1}\left(e^{-\alpha_1 t} - e^{-\alpha_2 t}\right) \text{(from b)} \quad (\alpha_2 \neq \alpha_1,\ t > 0)$$

P 3(e) $\alpha_2 = \alpha_1 = \alpha$

$$x_1 * x_2 = \int_0^t e^{-\alpha\tau} e^{-\alpha(t-\tau)}\, d\tau = e^{-\alpha t}\int_0^t e^0\, d\tau = t\, e^{-\alpha t}$$

END P3





Question 4: **response to a sinusoidal input**

In Q. P1 we sought the output of the RC circuit in Fig xxx for the case in which the input is a step function. This result was extended in Q. P2 and P3 for an impulse function input and for an exponential input. Now we examine the solution when the input is sinusoidal. This case is of special importance in this work as it opens the way to powerful tools for the solution of systems of much greater complexity than the introductory example under investigation here.

(a) Use the result in Q.P1(a) to show that

$$V_{in}(t) = RC.(dV_{out}/dt) + V_{out}(t) \qquad \text{Eqn. P4.1}$$

To simplify the analysis we will use the complex exponential A_in.exp(jwt) to represent the input sinusoid [recall that exp(jwt) = cos(wt) + j.sin(wt)].
In Q. P1(b) we obtained a solution of the DE by direct integration. However, sometimes it turns out that invoking a "feeling lucky" approach can provide the desired result:

a solution of the form

V_out = A_out . exp(j.phi_out) . exp(jw.t)

is substituted into the RHS in the above DE.
Show that this is a solution for a suitable value of A_out . exp(phi_out). (The suitable value is the one that makes the RHS = LHS). With A_in = 1, show that the sought value is

A_out . exp(phi_out) = 1/(1 + jwRC)

Hence show that V_out = V_in . 1/(1 + jwRC) = V_in . (1/RC)/ (jw + (1/RC))

Note that this result has a very interesting feature:

the output has the same form as the input.

[To discover the importance of this property it is worthwhile to think about the use of other waveforms to express the output in terms of the input. For example, a squarewave, a periodic ramp, a sawtooth (an optional lab exercise).]

(b) Use the result in (a) to obtain a formula for the ratio of output amplitude to input amplitude as a function of w for 1/RC = 1000 (rad/sec). Sketch the result, and find the value of w for which the ratio is 3dB.

SIGEx Expt. 10 RC in the time domain

Prep. solution Q. P4 (sinusoidal input)

(a) We return to the DE P.1.1 in P1(c):

$$V_{in}(t) = R.i(t) + V_{cap}(t) = R\,i(t) + \frac{1}{C}Q(t)$$

$$i(t) = \frac{dQ(t)}{dt} \implies V_{in} = R\frac{dQ}{dt} + \frac{1}{C}Q$$

$$V_{out} = V_{cap} = \frac{Q}{C} \implies Q = C\,V_{out}$$

$$\implies V_{in} = RC\frac{dV_{out}}{dt} + V_{out} \qquad \text{QED}$$

Now we proceed to solve for V_{out} when $V_{in} = A_{in}e^{j\omega t}$, and consider $V_{out} = A_{out}.e^{j\phi_{out}}.e^{j\omega t}$ as a candidate solution. Substituting this for V_{out} into the RHS of the DE gives

$$\text{RHS} = RC\,A_{out}\,e^{j\phi_{out}}.j\omega e^{j\omega t} + A_{out}e^{j\phi_{out}}e^{j\omega t}$$

$$= (j\omega RC + 1)\,A_{out}\,e^{j\phi_{out}}.e^{j\omega t}$$

It is evident that

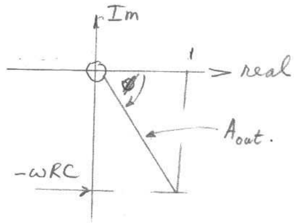
$$\text{RHS} = \text{Constant}_R . e^{j\omega t}$$
$$\text{LHS} = \text{Constant}_L . e^{j\omega t}$$

Hence the candidate solution for V_{out} satisfies the DE if $\text{Constant}_R = \text{Constant}_L$

i.e. $A_{out}\,e^{j\phi_{out}}(j\omega RC + 1) = A_{in}$.

For given RC (and $A_{in} = 1$, without loss of generality)

$$A_{out}\,e^{j\phi_{out}} = \frac{1}{1 + j\omega RC} = \frac{1 - j\omega RC}{1 + (\omega RC)^2}$$



$$\implies A_{out} = \left(1 + (\omega RC)^2\right)^{-\frac{1}{2}}$$

$$\phi_{out} = -\tan^{-1}\left(\frac{\omega RC}{1}\right)$$

SIGEx Expt10 RC in the Time Domain

Prep Solution Q.P4 (p.2/2)

(a) ctd.

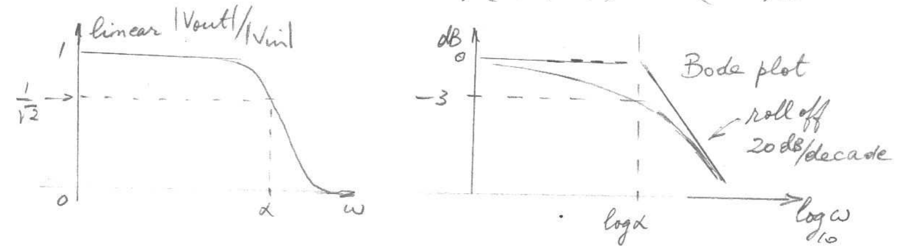
To obtain an expression for V_{out}/V_{in} we note that the RHS of the DE can be expressed as $(1+j\omega RC)V_{out}$

Hence,

$$V_{in} = (1+j\omega RC)\, V_{out}$$

i.e. $$\frac{V_{out}}{V_{in}} = \frac{(1/RC)}{(j\omega + (1/RC))}$$ QED.

(b) The ratio $|V_{out}|/|V_{in}| = \dfrac{\alpha}{\sqrt{(\omega^2+\alpha^2)}}$; $\left(\alpha = \frac{1}{RC} = 1000 \text{ rad/sec}\right)$



At $\omega = \alpha$ the amplitude ratio is $\frac{1}{\sqrt{2}}$ $(\alpha = 1000)$

This is $\frac{1}{\sqrt{2}}$ of the value at $\omega = 0$.

In dB this is $20\log_{10}(1/\sqrt{2}) = -3$ dB.

In the Bode plot representation of the gain vs frequency both axes are logarithmic.

When $\omega \gg \alpha$ $|V_{out}|/|V_{in}| \simeq \frac{\alpha}{\omega}$

Expressed in dB this is $20\log_{10}\alpha - 20\log_{10}\omega$.

Hence we have a straight line roll off asymptote with gradient 20 dB/decade

END P4

Question 5: **solution using the Laplace Transform**

(a) Look up the definition Y(s) of the Laplace transform of the function y(t). Show that the Laplace transform of (d/dt)y(t) is sY(s).
Solve Eqn P4.1 as a function of s by applying the Laplace transform to both sides (note that no restriction is imposed on the form of the input) .

Compare this result with the solution obtained with input

V_in (t) = A_in.exp(jwt)

Comment on similarities and differences.

(b) The transfer function is defined as V_out(s)/V_in(s). Use the result in (a) to write down the transfer function of the RC circuit.

(c) Find the Laplace transform of $y(t) = \exp(- a.t)$. Compare this with the transfer function in (b).

(d) What is the relationship between the transfer function and the impulse response that is apparent from (c)?

(e) On the basis of (d), what is the operation in the s domain that corresponds to convolution in the time domain? Confirm your answer by looking up the convolution theorem.

SIGEx Expt 10 RC in the time domain

Prep SOLUTION P5 (Laplace transform)

(a)

$$\mathcal{L}\{y(t)\} = Y(s) \triangleq \int_0^\infty y(t)\, e^{-st}\, dt$$

Laplace transform of the derivative:

$$\mathcal{L}\{y'(t)\} = \int_0^\infty e^{-st} \frac{d}{dt}(y(t))\, dt = \int_0^\infty e^{-st}\, dy(t)$$

Integrating by parts:

$$= e^{-st} y(t) \Big]_0^\infty - \int_0^\infty y(t)\, d\, e^{-st}$$

For s finite at $t = \infty$ $e^{-st} \to 0$ (subject to conditions on s)
at $t = 0$ $e^{-st} = 1$

Hence, $\mathcal{L}\{y'(t)\} = -y(0) + s\mathcal{L}\{y(t)\}$

For $y(0) = 0$ we have $\mathcal{L}\{y'(t)\} = sY(s)$

We now use this result to solve Eqn P4.1

$$V_{in}(t) = RC \frac{d}{dt}(V_{out}(t)) + V_{out}(t)$$

$$\Rightarrow V_{in}(s) = RC\, s V_{out}(s) + V_{out}(s)$$

$$\Rightarrow V_{out}(s) = \frac{V_{in}(s)}{RCs + 1}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = H(s) = \frac{\alpha}{s + \alpha}$$ [This is the result for (b)]

Comparison with result in P4: identical when $s = j\omega$.
This motivates the exploration of Laplace Transform theory as a tool for the analysis of systems like the RC circuit.

SIGEx Expt 10 RC in the time domain

Prep Solution P5 (p 2/2)

(c) $\mathcal{L}\{e^{-\alpha t}\} = \int_0^\infty e^{-\alpha t} e^{-st} dt = \int_0^\infty e^{-(s+\alpha)t} dt$

$= \frac{-1}{s+\alpha}\left[e^{-(s+\alpha)t}\right]_0^\infty = \frac{1}{s+\alpha}$ (subject to conditions on s)

This has the same form as H(s) in (b)

(d) The impulse response h(t) of the RC circuit is $\alpha e^{-\alpha t}$ (as obtained in P2). Applying the result in (c) gives $\mathcal{L}\{h(t)\} = \frac{\alpha}{s+\alpha}$

In (b) we found that the transfer function is given by $H(s) = \frac{\alpha}{s+\alpha}$

This is the same as the Laplace transform of the impulse response obtained above.

(e) In (a) we found that in the s domain Vout(s) is the product of the input $V_{in}(s)$ and the transfer function H(s).

In Q. P3(d) we found that in the time domain Vout(t) is the convolution of the input $V_{in}(t)$ and the impulse response h(t).

This suggests the idea that the Laplace transform of the convolution of two functions is the product of their individual Laplace transforms. Indeed this is known as the convolution theorem. The derivation is given in most standard texts.

END Q.P5 Solution.

Question 6: synthesized model of RC circuit

(a) Consider Eqn P4.1 in the Laplace domain, i.e.,

s.V_out = a . V_in + (- a) . V_out

Use the block diagram in Task 25 as a guide to model this equation using an integrator (1/s). Note that s.V_out(s) appears at the integrator input.

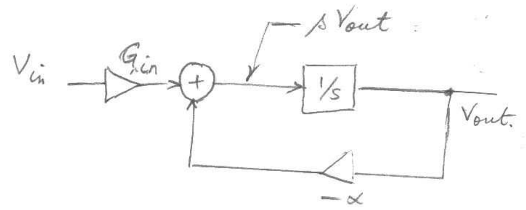
(b) In practical applications the use of a scaled integrator (k/s) may be necessary. Adjust the system equation so that the LHS is (s/k).V_out, and modify the model accordingly.

(c) Suppose k = 200 and a = 1000. Determine the corresponding value of a1 in the block diagram in Task 25.

SIGEx Expt 10 RC in the time domain

PREP SOLUTION P6 (frequency scaling)

(a)



Block diagram of system equation

G_{in} can be adjusted to suit the available amplitude range.

(b) The adjusted system equation is

$$\left(\frac{s}{k}\right)V_{out} = \left(\frac{\alpha}{k}\right)V_{in} + \left(-\frac{\alpha}{k}\right)V_{out}$$

In the above diagram

the integrator $\frac{1}{s}$ is replaced by $\frac{k}{s}$

the feedback gain $-\alpha$ becomes $-\frac{\alpha}{k}$

the adder output is $\frac{s}{k}V_{out}$.

(c) The new value of the coefficient a_1 is $\frac{1000}{200} = 5$

Note that a_1 is dimensionless. However k and α are in units of sec^{-1}. The feedback gain is negative.

END Q.P6

Question 7

How long will it take this RC NETWORK to rise to a level 37% below its final level ?

It should take 1 time constant = 1 ms

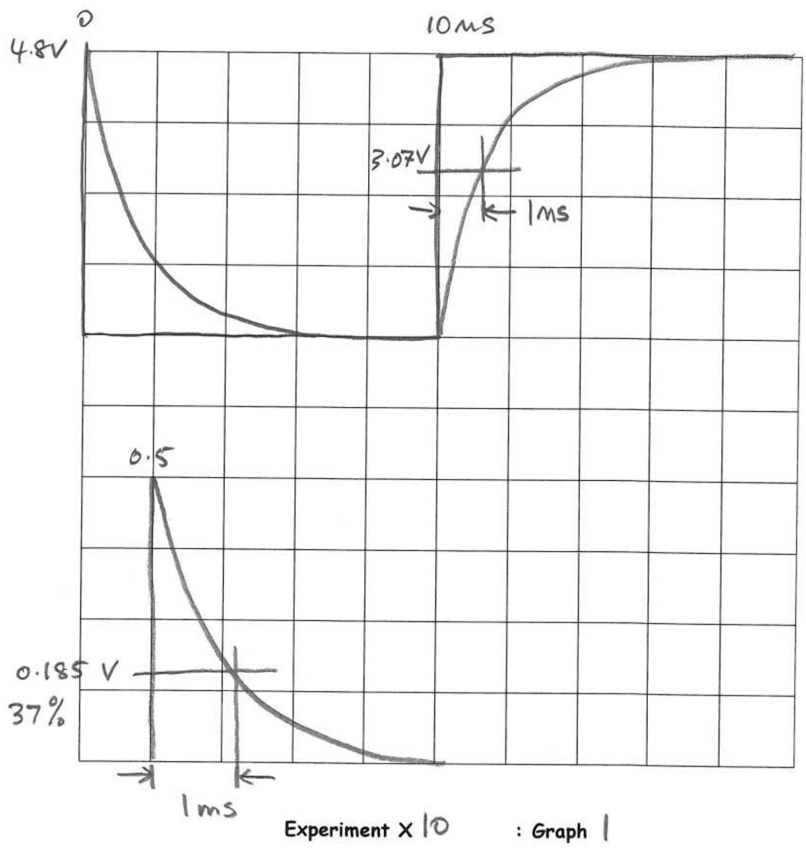
Question 8

Calculate the expected real circuit step response of the RC NETWORK using the real circuit values and real circuit input values. These values are available in the User Manual. For your convenience they are R=10kohm, and C=100nF

1 ms

Measured response corresponds with theory.

SIGEx Lab Manual Vol.1 : Instructors Manual.



Experiment X 10 : Graph 1

Graph 1: step and impulse responses

Question 9
What is the width if the impulse. What is its maximum amplitude ?

0.1 ms; 4.8V

Question 10

What is the equation for the measured impulse response using actual circuit values ? How does this compare with theory ?

See first part of answer for Q 11

Question 11

Explain why the impulse response reaches the peak value that it does.

HINT: superposition of 2 step responses is involved.

Finite impulse considered to be the sum of 2 step responses separated by 0.1ms. Hence the

response to 1st step is $1 - e^{-t/RC}$; for t>0.

Response to 2nd step is $-[1 - e^{-(t-0.0001)/RC}]$; for t>0.0001; a time delayed response.

Overall impulse response is the sum of these (with 2nd response = 0 until t=0.0001).

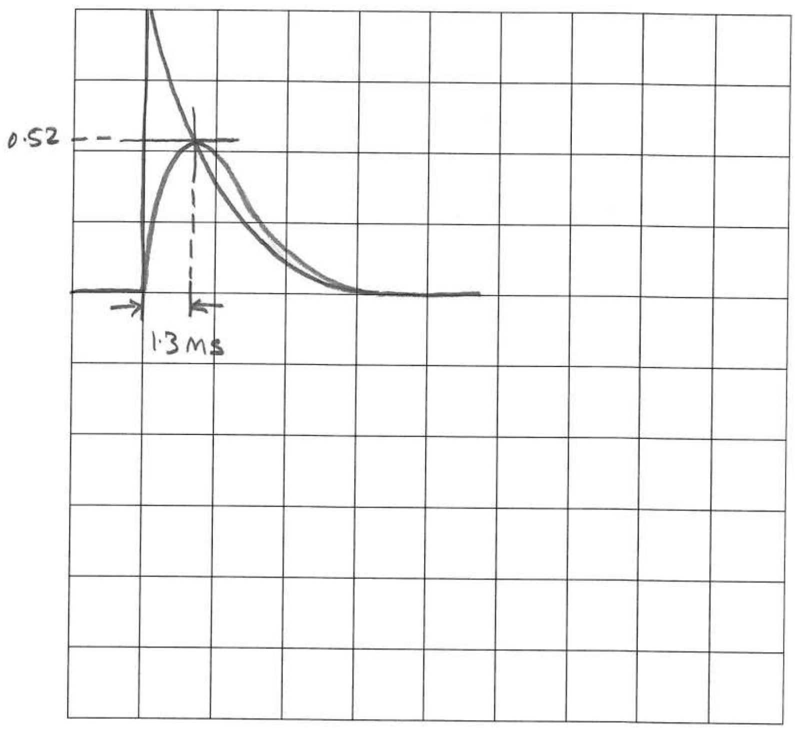
The peak is value of 1st response at t=0.0001 ie: $1 - e^{-0.0001/0.001} = 0.0952$ (normalized)

Hence = 0.0952 x 4.8 = 0.457 denormalized.

Question 12

What is the equation for the output signal and how does it compare with the theoretical output expected from this network ? Refer to your work in preparation question 3.

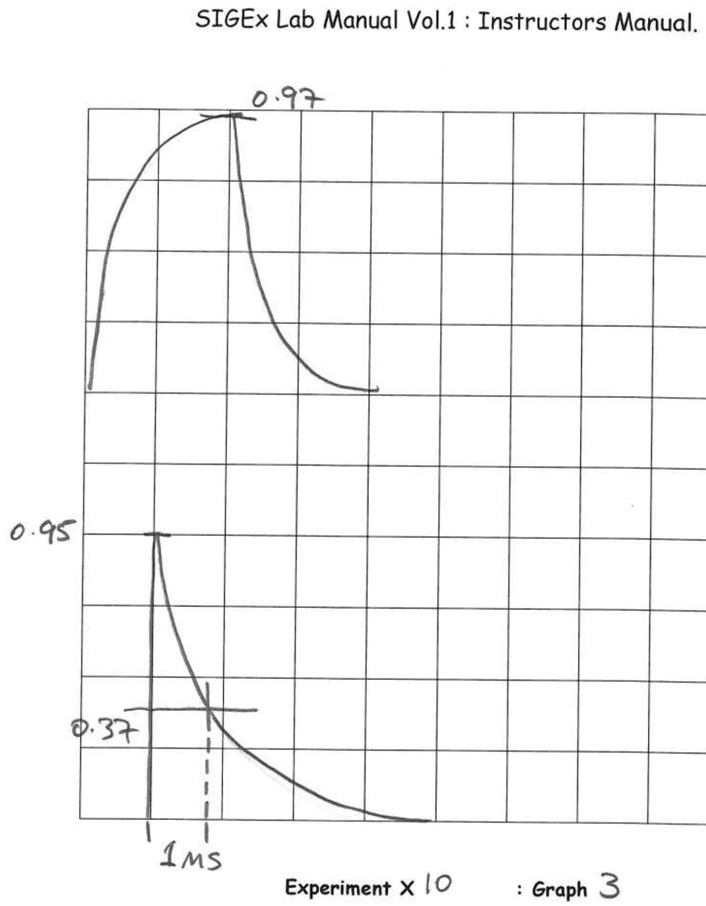
See prep.3



Experiment x 10 **: Graph** 2

Graph 2: exponential pulse response

Question 13
Show that RC = | 1/(k.a1)|; where |k.a1| = 1000



Graph 3: step and impulse response of synthesized system

Question 14
What values of a0 and a1 have you found give your synthesised system a perfect match to the actual RC NETWORK ?

a0= + 0.84; a1 = - 0.07

Question 15

What is the signal at the input to the integrator ? Is this expected ? Explain:

It is the differential of the output.

Yes,

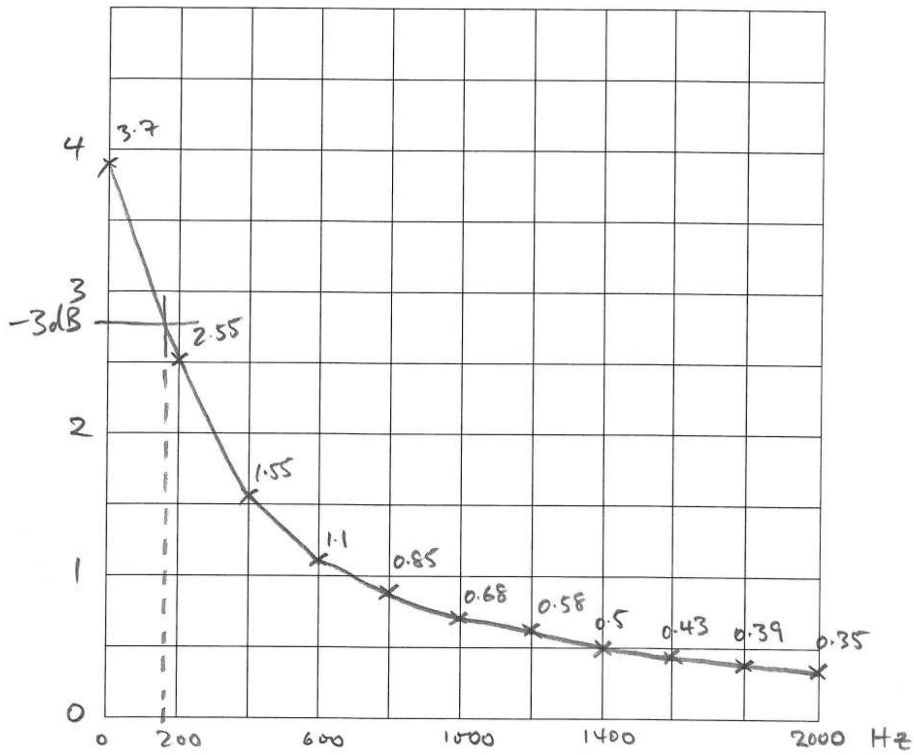
Question 16

Using the measured values above, what is the actual transfer function for your synthesised network which matches the actual RC network ? Show your working.

k measured as 11927.

Question 17

Explain any discrepancies you find between expected theory and measurements. What sources of error are responsible for these ?



Experiment X 10 : Graph 4

Graph 4: frequency response and bode plot of synthesized system

Name: ____________________________________

Class: ____________________________________

X11 - Poles and zeros in the Laplace domain

SIGEx

Pre-requisite work

Preparation will be required in order for the hands-on lab work to make sense. This guided preparation is a revision of theory you will have covered in lectures and is presented below as a number of computation exercises. This work should be completed before attempting the lab.

Question 1

For the system in Figure 1, obtain a differential equation relating the output $x_0(t)$ and the input $u(t)$. Show by substitution that $x_0 = e^{j\omega t}$ is a solution and determine the corresponding input $u(t)$ *that produces this output.*

SIGEx11: Poles and zeros in the Laplace Domain

18-3-2011V (per Carlo mail 16/3)

Solutions to prep questions.

Question 1 Set up the 2nd order D.E. for Fig1.

$$x_1 = \frac{dx_0}{dt} \qquad x_2 = \frac{dx_1}{dt} = \frac{d^2x_0}{dt^2}$$

$$x_2 = u - a_0 x_0 - a_1 x_1$$

$$\Rightarrow u = x_2 + a_1 x_1 + a_0 x_0$$

$$= \frac{d^2x_0}{dt^2} + a_1\frac{dx_0}{dt} + a_0 x_0$$

This is the required D.E. for $x_0(t)$.

Substitute $\exp(j\omega t)$ for x_0:

$$R.H.S. = (j\omega)^2 x_0 + (j\omega)a_1 x_0 + a_0 x_0$$

$$= x_0\left[(j\omega)^2 + a_1(j\omega) + a_0\right] = x_0 C$$

where C is not a function of t, i.e. it is a constant determined by the given values of a_0, a_1 and ω.

Hence $x_0 = \exp(j\omega t)$ is a solution when

$$u = C \cdot \exp(j\omega t).$$

If you are uncomfortable with a complex-valued function to represent the behaviour of a system that is supposed to operate with real-valued signals, $x_0 = \cos(\omega t)$ or $\sin(\omega t)$ could be used. However, you will quickly discover that the exponential function has a very useful property that simplifies the math considerably. Remembering that $\cos(\omega t)$ is Re{ $e^{j\omega t}$}, you can carry out the analysis with $e^{j\omega t}$ then simply take the real part of the result. Practitioners generally don't bother with the formality of taking the real part. Moreover, complex valued signals are easily realized in digitally implemented systems, and indeed, frequently used, for example in modulators and demodulators of dial-up modems.

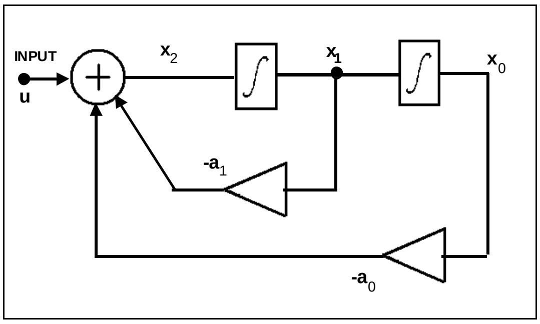


Figure 1: schematic of 2nd order integrator feedback structure without feedforward.

Question 2

From the above, with $x_0 = e^{j\omega t}$, obtain an expression for the ratio x_0/u as a function of $j\omega$ (not just "ω"; the reason for this will emerge shortly). Note that this ratio is complex valued. Then, obtain its magnitude and phase shift as functions of ω (not $j\omega$).

SIGEx11 solutions P-Z in Laplace

Prep Question 2

From Question 1, ratio of output x_o to input u

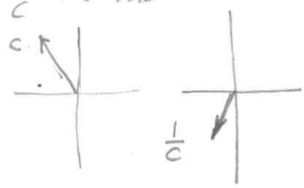
$$\frac{x_o}{u} = \frac{1}{C} = \frac{1}{(j\omega)^2 + a_1(j\omega) + a_o}$$

$$\left|\frac{x_o}{u}\right| = \frac{1}{|C|} = \frac{1}{\sqrt{(a_o - \omega^2)^2 + a_1^2\omega^2}}$$

$$\angle\left(\frac{x_o}{u}\right) = -\tan^{-1}\left(\frac{a_1\omega}{a_o - \omega^2}\right)$$

N.B. In the formula for $\angle\left(\frac{x_o}{u}\right)$ based on $\tan^{-1}$ there is a residual ambiguity of 180°. This is readily resolved by plotting $\frac{1}{C}$ in the Complex plane.

For example $C = -\frac{3}{5} + j\frac{4}{5}$



Details for the above

Denominator $C = (a_o - \omega^2) + j\cdot a_1\omega$

$$|C|^2 = (a_o - \omega^2)^2 + a_1^2\omega^2$$

$$\angle C = \tan^{-1}\left(\frac{a_1\omega}{a_o - \omega^2}\right)$$

$$\Rightarrow \left|\frac{x_o}{u}\right| = \frac{1}{|C|} = \frac{1}{\sqrt{(a_o-\omega^2)^2 + a_1^2\omega^2}}$$

$$\angle\left(\frac{x_o}{u}\right) = -\angle C$$

Question 3

From the results in Question 1 above, plot the magnitude $|x_0/u|$ versus ω (radians/sec) for the case $a_0 = 0.81$, $a_1 = 0.64$. Note that there is a progressive fall off as ω increases. Hence, we can think of this system as realizing a lowpass filter.

Question 4

We now consider an alternative way of getting the response. With a little algebra we create a graphical medium that will provide an intuitive environment for visualizing and generating both magnitude and phase responses.

First, return to the expression for x_0/u obtained in (a) and replace "$j\omega$" by the symbol "s". Look upon s merely as a convenient stand in for $j\omega$. It is not necessary to ascribe any deeper significance to this substitution for the purposes of this lab. The result is the (complex-valued) rational function

$$x_0/u = H(s) = 1/(s^2 + a_1.s + a_0) \qquad \text{(Eqn 1).}$$

For the case $a_0 = 0.81$, $a_1 = 0.64$ (as in (b), express the denominator quadratic in the factored form $(s - p_1)(s - p_2)$, where p_1 and p_2 are the roots. Show that these are given by

$$p_1 = 0.9(\cos(110.8°) + j.\sin(110.8°)) = 0.9\exp^{j0.616\pi}$$
$$p_2 = 0.9(\cos(110.8°) - j.\sin(110.8°)) = 0.9\exp^{-j0.616\pi} \qquad \text{(Eqn 2).}$$

Pre-lab preparation work

SIGEx11 Solutions P-Z in Laplace domain

Prep Q. 4.

We seek to express the roots p_1 and p_2 in terms of a_0 and a_1. To simplify the math we exploit the theorem about complex roots of polynomials with real coefficients, namely that complex roots occur in conjugate pairs. In this example we have $p_2 = p_1^*$. (The proof is straightforward and is given in the addendum below)

Multiplying $(s-p_1)(s-p_2)$ we have $s^2 - (p_1+p_2)s + p_1 p_2$

Equating coefficients:

$$p_1 p_2 = p_1 p_1^* = |p_1|^2 = a_0 \Rightarrow |p_1| = \sqrt{a_0}$$

$$-(p_1+p_2) = -(p_1+p_1^*) = -2Re(p_1) = a_1 \Rightarrow Re(p_1) = -\frac{a_1}{2}$$

Express p_1 in polar form: $p_1 = Ae^{j\theta}$.

Hence $A = |p_1| = \sqrt{a_0}$

$$A\cos\theta = Re(p_1) = -\frac{a_1}{2} \Rightarrow \cos\theta = -\frac{a_1}{2\sqrt{a_0}}$$

With $a_0 = 0.81$ and $a_1 = 0.64$

$$A = \sqrt{a_0} = 0.9 \qquad \cos\theta = -\frac{0.64}{2\times 0.9} \Rightarrow \theta = 110.8^\circ$$

Conditions for complex roots:

Roots are complex valued when A is real and $|\cos\theta| < 1$

Hence $a_0 > 0$ and $\left|\frac{a_1}{2\sqrt{a_0}}\right| < 1$ i.e. $|a_1| < 2\sqrt{a_0}$

SIGEx 11 $\boxed{\text{prep Q4 addendum}}$ P-Z in Laplace

This addendum is the proof for $p_2 = p_1^*$ when a_0 and a_1 are real valued.

Equating coefficients we have

$$-(p_1 + p_2) = a_1 \qquad p_1 p_2 = a_0$$

We assume that the conditions for complex roots are satisfied.

Let $p_1 = A_1 e^{j\theta_1} \qquad p_2 = A_2 e^{j\theta_2}$.

$\Rightarrow p_1 p_2 = A_1 A_2 e^{j(\theta_1 + \theta_2)} = a_0$

Since a_0 is real, $\theta_1 + \theta_2 = 0 \Rightarrow \theta_2 = -\theta_1$

We now show that $A_2 = A_1$:

$-(p_1 + p_2) = -(A_1 e^{j\theta_1} + A_2 e^{-j\theta_1}) = a_1$

Since a_1 is real,

$Im(p_1 + p_2) = 0$, i.e. $A_1 \sin\theta_1 + A_2 \sin(-\theta_1) = 0$

$\Rightarrow A_1 - A_2 = 0 \Rightarrow A_2 = A_1 \quad [\sin\theta_1 \neq 0]$

Hence $p_2 = p_1^*$

Question 5

Express the complex points p_1 and p_2 from equation 2 above as the non-exponential complex form of a + ib, that is, with a real and imaginary part.

SIGEx 11 Solutions p-2 in Laplace domain

Pre-lab preparation work

new

Question 5

Express the complex points p_1 and p_2 from equation 2 above as the non-exponential complex form of a + ib, that is, with a real and imaginary part.

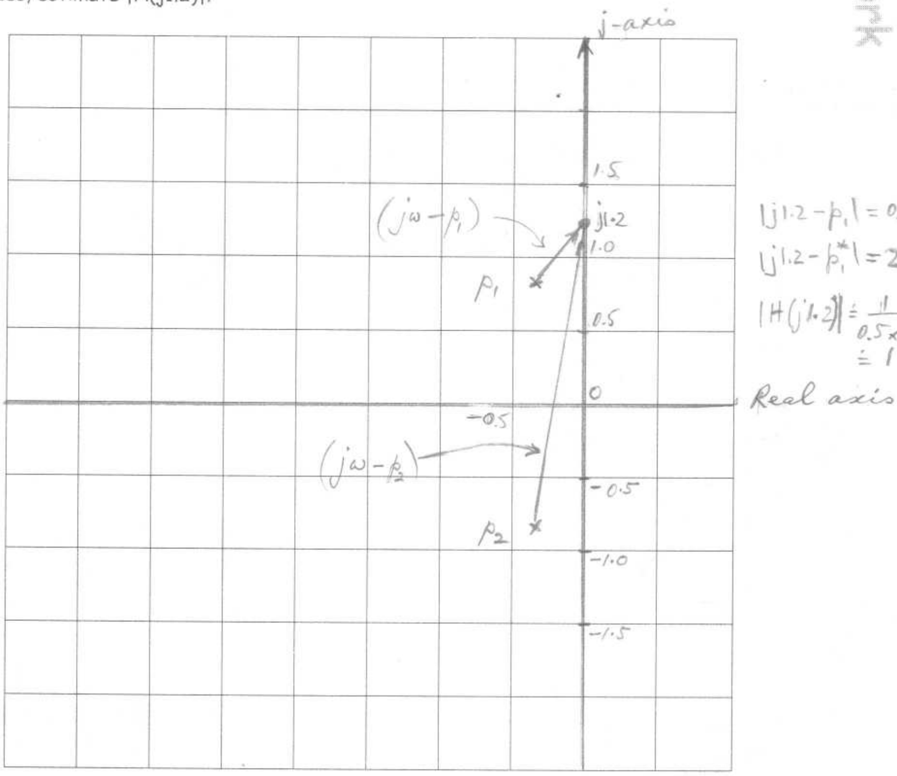
From Q4: Q4 $0.9\cos(110.8^\circ) \pm j0.9\sin(110.8^\circ) = -0.320 \pm j0.8412$

From quadratic: $-\left(\frac{a_1}{2}\right) \pm j\sqrt{a_0 - \left(\frac{a_1}{2}\right)^2}$

d)

Question 6

Next, we look at a graphical approach for evaluating the factors ($s - p_1$) and ($s - p_2$). Place crosses ("x") on a complex plane at the locations corresponding to p_1 and p_2, as obtained in (c) above. Place a dot at the point 1.2 on the j axis, i.e., the complex value $j\omega = j1.2$. Join this point and the crosses at p_1 and p_2 with straight lines. Satisfy yourself that the lengths of these joining lines are $|j\omega - p_1|$ and $|j\omega - p_2|$. Noting that $1/|H(j\omega)|$ is the product of these two magnitudes, estimate $|H(j1.2)|$.



(0.48)

$|j1.2 - p_1| = 0.5$

$|j1.2 - p_1^*| = 2$ (2.07)

$|H(j1.2)| \doteq \frac{1}{0.5 \times 2} \doteq 1$

Graph 2: vector subtraction

Experiment 11 – Poles & zeros in the Laplace domain © 2011 Emona Instruments 11-5

2.666

Question 6

Next, we look at a graphical approach for evaluating the factors ($s - p_1$) and ($s - p_2$). Place crosses ("x") on a complex plane at the locations corresponding to p_1 and p_2, as obtained in (c)

above. Place a dot at the point 1.2 on the j axis, i.e., the complex value $j\omega = j1.2$. Join this point and the crosses at p_1 and p_2 with straight lines. Satisfy yourself that the lengths of these joining lines are $|j\omega - p_1|$ and $|j\omega - p_2|$. Noting that $1/|H(j\omega)|$ is the product of these two magnitudes, estimate $|H(j1.2)|$.

Question 7

Use the idea above to obtain estimates of $|H|$ at other frequencies and thus produce a sketch graph of $|H|$ over the range 0 to 5 radian/s. (ie: ω will range from 0 to 5). Notice that the presence of a peak in the response is obvious from the behaviour of the vector from p1 as the dot on the j axis is moved near p1. Note that this vector has much greater influence than the other vector, especially near the peak. Compare this estimate with the computed result you obtained in (b). Plot at least 4 points over this range, choosing your points to reflect the important characteristics of this response.

Explain why the vector from p1 has a greater influence on the peak of the response.

Re Question 7:

The procedure is the same as in Q6. The main purpose of the exercise is to demonstrate how the general shape of the response can be estimated intuitively from the position of the poles in the the s plane. A secondary aspect is to compare the outcome with the exact result in Q3.

The reason why the pole in the lower half plane usually has less influence is because the rate of change of its contribution is relatively small as the position of the frequency point on the j axis moves closer to the upper half plane pole.

Question 8

The roots p_1 and p_2 of the denominator polynomial of $H(s)$, marked as crosses on a plane of the complex variable s are known as *poles* of $H(s)$. Note that in the example case, p_2 is the complex conjugate of p_1. Why is this so?

Question 9

Derive Eqn1 from the schematic (block) diagram, Figure 1, without using the differential equation step. That is, treat the integrator as a "gain" of value 1/s and process the equations as algebra.

SIGEx 11 prep solutions : Q9–11

Q9 Solution

$x_1 = s x_0$

$x_2 = s x_1$

$x_2 = u - a_1 x_1 - a_0 x_0$

$\Rightarrow s^2 x_0 + a_1 s x_0 + a_0 x_0 = u$

$\Rightarrow x_0 = u/(s^2 + a_1 s + a_0)$

Q10 Solution

$y = b_2 x_2 + b_1 x_1 + b_0 x_0$

$= b_2 s^2 x_0 + b_1 s x_0 + b_0 x_0$

$= x_0 (b_2 s^2 + b_1 s + b_0)$

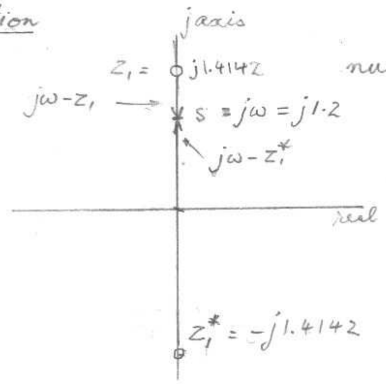
$\Rightarrow \frac{y}{u} = (b_2 s^2 + b_1 s + b_0)/(s^2 + b_1 s + b_0)$

With $b_0 = 2$, $b_1 = 0$, $b_2 = 1$

numerator $= s^2 + 2$

$\Rightarrow$ roots $z_1 = j1.414$ $\quad z_2 = -j1.414$.

Q 11 Solution



numerator $= |(j\omega - z_1)(j\omega - z_1^*)|$

$= (1.4142 - 1.2)(1.4142 + 1.2)$

$= (0.2142)(2.6142)$

$= 0.560$

Question 10

Next we proceed to the system in Fig 2. Note that this is a simple extension of the feedback only system in Fig. 1. Use Eqn 1 to obtain the output/input equation y/u ,

$$y/u = H\_y(s) = (b_2.s^2 + b_1.s + b_0)/(s^2 + a_1.s + a_0) \qquad \text{Eqn3}$$

(i) Consider the case $b_0 = 2.0$, $b_1 = 0$, $b_2 = 1.0$. Show that the roots of the numerator for these coefficients are $z_1 = 0 + j1.414$, $z_2 = 0 - j1.414$. Place an "o" on these points on the same s plane diagram you used to mark the poles, Graph 2. The roots of the numerator are known as "*zeros*".

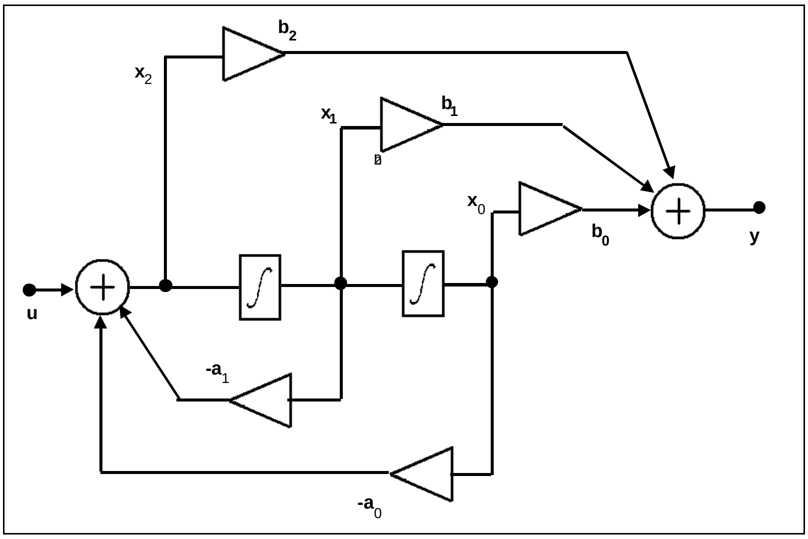


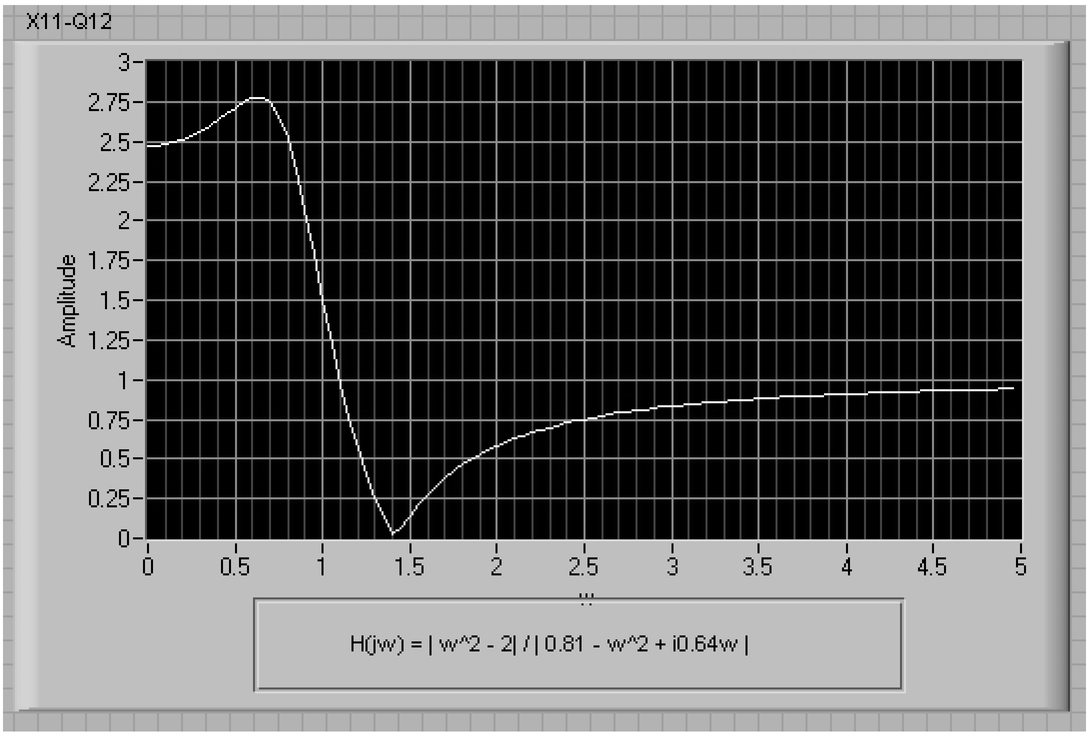
Figure 2: schematic of 2nd order integrator feedback structure with feedforward combiner.

Question 11

Using the zeros with the method from Question 6, carry out the graphical estimation of the numerator of Eqn 3 at s = j1.2. Note that this is a special case, with the zeros located on the j axis (since $b_1 = 0$). Hence, the lines joining the point jw and the zeros will lie on the j axis. Combine the numerator and denominator estimates to obtain |H_y(j2)|. Extend to other values of w, and sketch the magnitude response |H_y(jω)|. Comment on the presence of a null at ω = 1.414.

Question 12

(optional) Compute |H_y(jω)| from Eqn 3 and assess the quality of the estimate based on poles and zeros.



Question 13

With a_0 and a_1 as in Question 3, apply the pole-zero method to obtain approximate graphs of the magnitude response for the following cases:

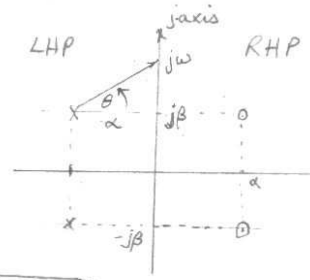
$b_1 = 1, b_0 = b_2 = 0$
$b_2 = 1, b_0 = b_1 = 0$
$b_2 = 1, b_1 = -a_1, b_0 = a_0$
$b_2 = 1, b_1 = 0, b_0 = a_0$

State the name of the response type corresponding to each case (e.g., bandstop, allpass, etc). For the allpass case, plot the phase and/or group delay response (group delay = $-$d(phase)/dω). Find out and note here an application for the allpass response.

$b_1 = 1, b_0 = b_2 = 0$	=> Bandpass (zero at s=0)
$b_2 = 1, b_0 = b_1 = 0$	=> Highpass (double zero at s=0)
$b_2 = 1, b_1 = -a_1, b_0 = a_0$	=> Allpass (mirror zeros in RHP)
$b_2 = 1, b_1 = 0, b_0 = a_0$	=> Notch (zeros on j-axis opposite poles)

SIGEx 11 Solutions P.Z in Laplace domain

Prep Q13: Group delay for allpass case



The biquad allpass transfer function has two poles and two zeros in conjugate pairs. The zeros are symmetrically opposite the poles in the right half plane.

NB This analysis assumes complex poles & zeros

However allpass can be realized with real poles and zeros

Using the phasor method introduced in Q.6 and Q.7 it is evident that the magnitudes of the transfer function numerator and denominator are equal for any ω.
Hence $|H(j\omega)| = 1$ for all ω.
Thus, allpass are useful to modify phase responses without affecting the magnitude response. The group delay is an alternative approach to represent phase response. It is defined as

$$\tau(\omega) = -\frac{d}{d\omega}(\measuredangle H(j\omega))$$

The overall phase response is

$$\measuredangle H(j\omega) = \measuredangle \text{numerator} - \measuredangle \text{denominator}.$$

with $\measuredangle$ denominator $= \measuredangle (j\omega - p_1) + \measuredangle (j\omega - p_1^*)$
and $\measuredangle$ numerator $= \measuredangle (j\omega - z_1) + \measuredangle (j\omega - z_1^*)$

Each of these four contributions has the same form, hence we will derive the group delay corresponding to $\measuredangle (j\omega - p_1)$ and use the result to obtain the other three.
An interesting outcome is that unlike the phase response, the group delay expression is algebraic, it does not have inverse trig functions.

→ p 2/2

Prep Q13: Group delay ctd (p2/2)

Group delay contribution of factor $(j\omega - p_1)$

From the s plane phasor diagram we have

$$p_1 = \alpha + j\beta \implies \tan\theta(\omega) = \frac{\omega - \beta}{\alpha}$$

where $\theta(\omega)$ is $\angle(j\omega - p_1)$ [NB! different to θ in Q4]

Differentiate both sides:

LHS: $\frac{d}{d\omega}(\tan\theta(\omega)) = (\sec^2\theta).\frac{d\theta}{d\omega} = (1+\tan^2\theta)\frac{d\theta}{d\omega}$

RHS: $\frac{d}{d\omega}\left(\frac{\omega-\beta}{\alpha}\right) = \frac{1}{\alpha}$

Hence $\tau_{p_1}(\omega) = -\frac{d\theta}{d\omega} = \frac{-(1/\alpha)}{1+\left(\frac{\omega-\beta}{\alpha}\right)^2} = \frac{-\alpha}{\alpha^2+(\omega-\beta)^2}$

and $\tau_{p_1^*}(\omega) = \frac{-\alpha}{\alpha^2+(\omega+\beta)^2}$

$$\tau_{poles}(\omega) = \tau_{p_1} + \tau_{p_1^*} = \frac{(-\alpha)}{\alpha^2+(\omega-\beta)^2} + \frac{(-\alpha)}{\alpha^2+(\omega+\beta)^2}$$

(From Wolfram Alpha)

$$= \frac{-2\alpha F}{(F-2\beta\omega)(F+2\beta\omega)} = \frac{-2\alpha F}{F^2-(2\beta\omega)^2}$$

where $F = \alpha^2+\beta^2+\omega^2 = a_0+\omega^2$ (since pole magnitude^2 $= \alpha^2+\beta^2$ $= a_0$)

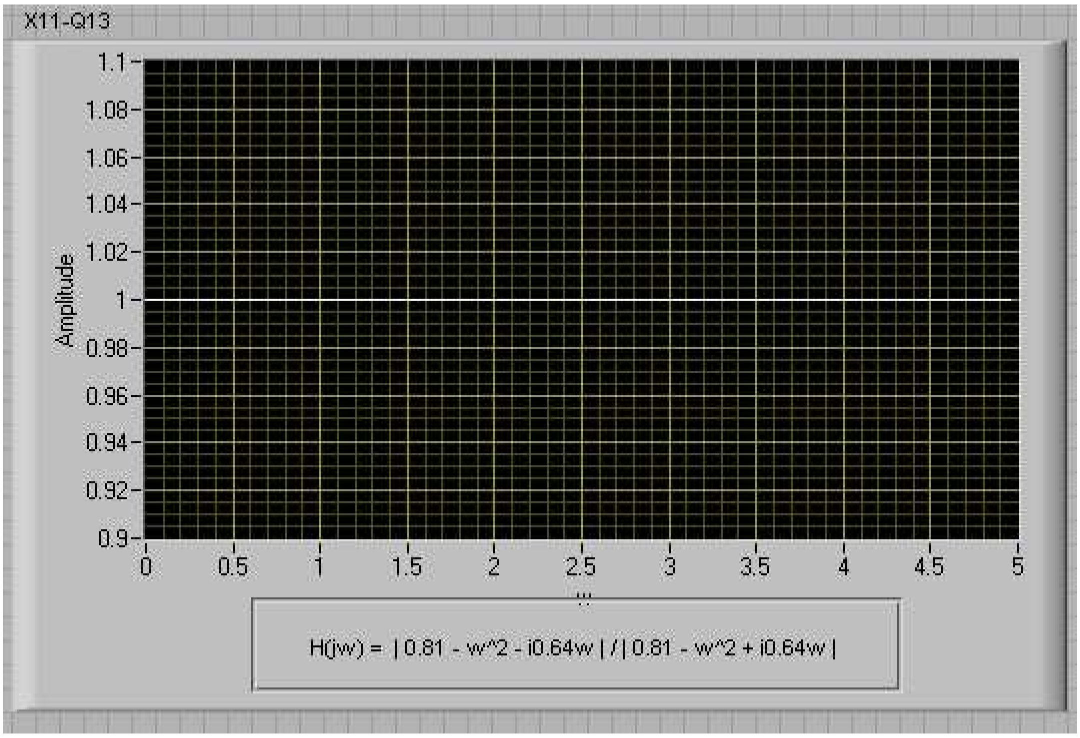
It is readily shown that

$$\tau_{zeros}(\omega) = -\tau_{poles}(\omega)$$

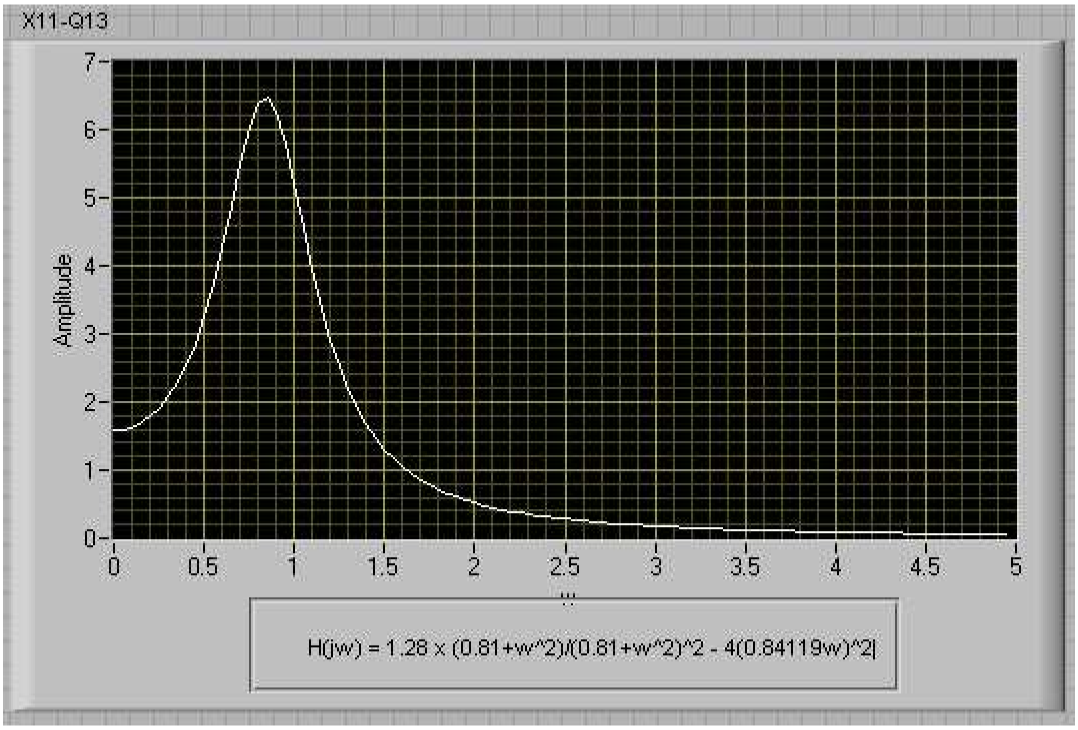
Hence

$$\tau(\omega) = \tau_{zeros} - \tau_{poles} = \frac{-4\alpha(a_0+\omega^2)}{(a_0+\omega^2)^2-(2\beta\omega)^2}$$

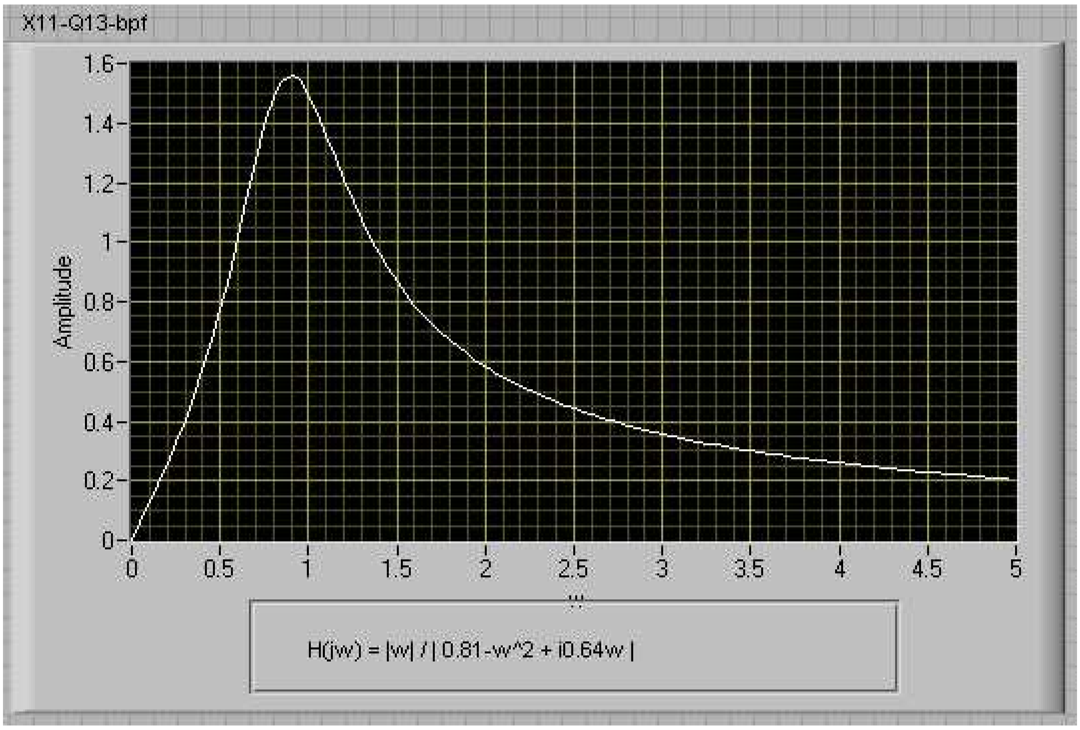
N.B. In the above expression $\alpha < 0$ (since $a_1 > 0$).
Formulas for α and β in terms of coefficients a_0 and a_1 are given in the solution of Q5



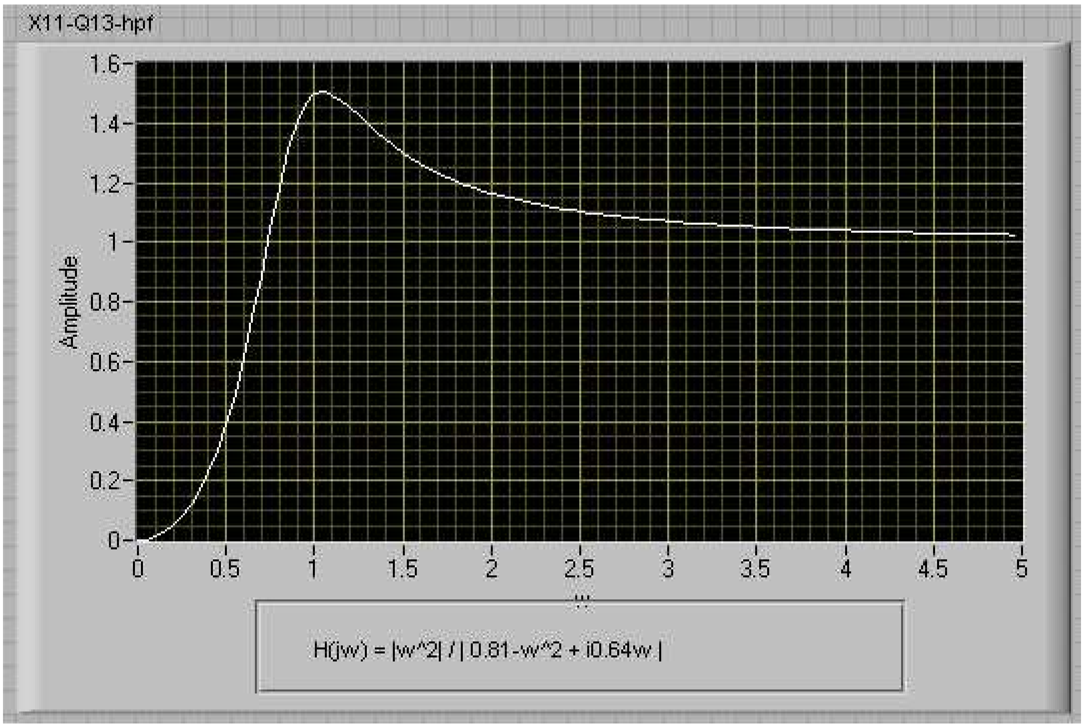
$b_2 = 1$, $b_1 = -a_1$, $b_0 = a_0$ => Allpass (mirror zeros in RHP)



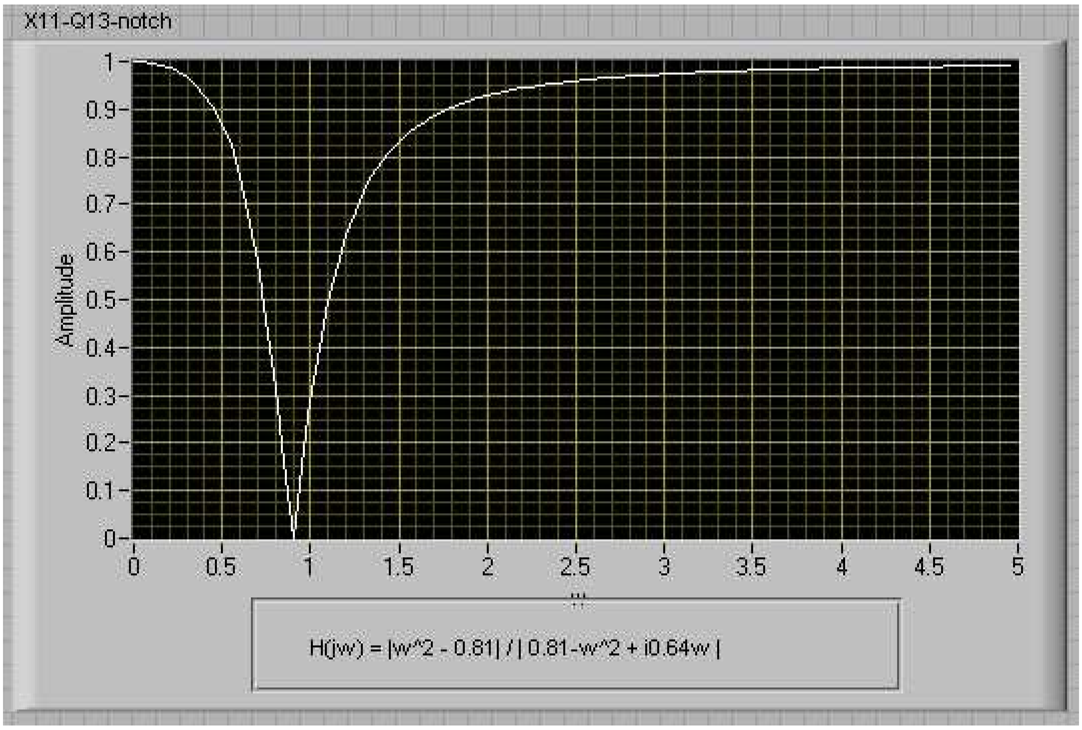
(group delay = $-$d(phase)/dω).



$b_1 = 1$, $b_0 = b_2 = 0$ => Bandpass (zero at s=0)



$b_2 = 1$, $b_0 = b_1 = 0$ => Highpass (double zero at s=0)



$b_2 = 1$, $b_1 = 0$, $b_0 = a_0$ => Notch (zeros on j-axis opposite poles)

Question 14

The integrators in Figs 1 and 2 were depicted as having unity gain. A practical realization normally has an associated gain constant. The corresponding integrator equations have the form

$$x_0 = k \cdot \int (x_1)\, dt$$
$$x_1 = k \cdot \int (x_2)\, dt$$

Note that k is not dimensionless. Its unit is sec^{-1}. The SIGEx INTEGRATOR modules provide a choice of four values of k, selectable by means of on-board switches. The switches are labelled "INTEGRATION RATE" and the selection and associated value is displayed on the SIGEx SFP. Suppose k = 12,500 sec^{-1} is selected. Modify the frequency scale for the response in (b) above to reflect this choice of k. Explain your reasoning here.

SIGEx 11 prep solutions: Q14

Proceed as in Q9 with s/k replacing s

The new equations are

$$x_1 = (s/k)\, x_0$$
$$x_2 = (s/k)\, x_1$$

$$\Rightarrow \left(\frac{x_0}{u}\right) = H(s) = 1/\left((s/k)^2 + a_1 (s/k) + a_0\right)$$

With $s = j\omega$ $\quad |H(j\omega)| = \left|1/\left((j\frac{\omega}{k})^2 + a_1 (j\frac{\omega}{k}) + a_0\right)\right|$

From this we see that the introduction of the integrator gain k leads to a change in the frequency scale. An easy way to understand the effect is to consider the frequency of the peak. The magnitude response in Graph 1 (Q3) corresponds to k = 1 and has its peak at ω = 0.8 rad/sec. When k = 12,500 sec^{-1} the frequency of the peak is $0.8 \times 12{,}500 = 10{,}000$ rad/sec.

Hence, the integrator gain constant k upscales the frequency axis by a factor of k, i.e. 12,500 in this instance.

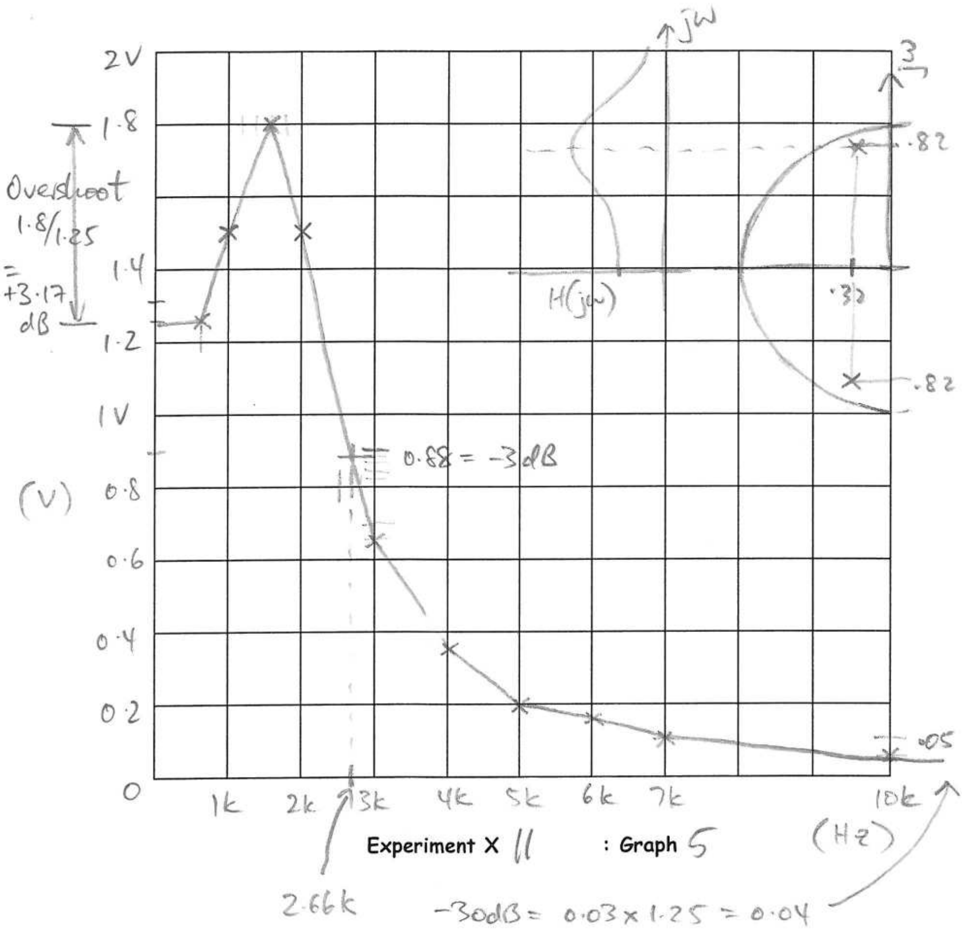
Question 15
Measure and plot the gain frequency response at the output of the second integrator (x_0) onto Graph 5. Confirm that this is a lowpass response similar to the theoretical predictions you obtained in prep item (Q3) (the rescaling of the frequency axis will be calculated next). Note the -3dB cut-off frequency and the frequency at which the response drops to -30dB. Measure the overshoot (if any) and note the frequency of the peak.

DC gain=1.25; F-3db=2.67kHz; f-30db=3.75dB; fpk=1.6kHz @ +3dB

Question 16
Calculate the integration rate as (rise(V)/run(s)) / input voltage (V). The units for integration rate are sec^{-1}. Repeat your measurement for a falling ramp and confirm that the magnitudes are equal. Compute rates for all 4 switch positions in case you need this information later on.

@1kHz.....UP/UP: 6.3V/0.5ms/1 = 12600; UP/DOWN: 9.6V/0.5ms=19200

DOWN/UP: 16V/0.5ms = 32000; DOWN/DOWN: 21/0.05ms=420,000 @ 10kHz



$-30dB = 0.03 \times 1.25 = 0.04$

$f_{-30dB} => 10kHz$

$1 + 0.6K/s + 0.81/s^2 \Rightarrow (-0.32 \pm 0.84i)$

When $j\omega = .84$, $H(s) = max.$

$\therefore f = \frac{.84}{2\pi} = 0.133 \times 12,600$

$= 1684\ Hz.$

Graph 1: poles and magnitude response

Question 17

Measure the frequency of the response peak, the 3dB frequencies, and hence, the 3dB bandwidth.

Fpk=1.8kHz @ 1.5V

f-3db=1.5x0.707=1.06V; f@1.06V=1.28khz & 2.74kHz.....BW=1.46kHz

Question 18

Calculate the geometric and arithmetic means of the 3dB frequencies. Compare this with the peak frequency. Consider which of these gives the closer agreement. This is not easy to resolve as the peak is quite flat, and pinpointing it can be challenging. It turns out that for this type of second-order system the peak is at the geometric mean of the 3dB frequencies (see Tut Q.2). Since these can be measured more accurately, this provides a better alternative for measuring the resonance frequency. From Tut Q.2 it is readily shown that this formula is not restricted to a 3dB bandwidth criterion. You may like to put this to the test, e.g. for the 6dB frequencies.

Arithmetic mean = (1.28k + 2.74k)/2 = 2.01kHz

Geometric mean = sqrt(1.28k x 2.74k) = 1.87kHz

Question 19

In Tut Q.2 it is shown that the bandpass response peak is at ($\sqrt{a_0}$) rad/sec. Using this formula and measurement results obtain an alternative estimate of the scaling factor, and compare this with the results of the integrator gain measurements in T1.3. Consider which of these is the more reliable .

Record these results for use in Tut Q.2.

Fpk=1.6k = 0.9/2pi x IG..........hence, IG = 11,170

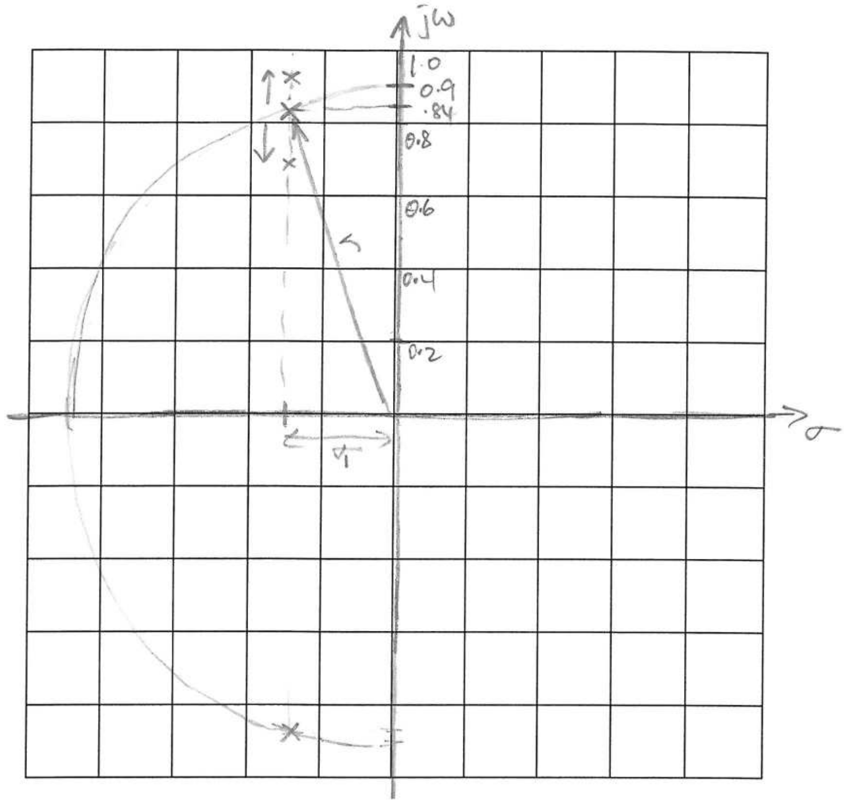
11,170/12600 = ...12% difference...measurement errors ?

Question 20

Consider practical uses of these properties and record your comments.

Convenient for tuneable digital filters

For fine tuning responses



Experiment X 11 : Graph 6

$a_1 = -0.64 \therefore \sigma_1 = a_1/2 = -0.32$

When $a_0 = -0.81$, $r = 0.9$

$a_0 = -0.91$, $r = .954 \Rightarrow r\uparrow, \sigma$ const. $\therefore f_{pk}\uparrow$

$a_0 = -0.7$, $r = .836 \Rightarrow r\downarrow, \sigma$ const $\therefore f_{pk}\downarrow$

When $a_0 = -0.81$, $r = 0.9$, as $a_1\uparrow$, less peak gain

" " " , as $a_1\downarrow$, more peak gain

Graph 2: locus of poles

Question 21
Record your value of a_1 here as you will need it later.

a_1 = -0.4, gives several cycles as per Fig 7, (other values are equally valid)

Question 22
Record your observations.

As a_1 tends to 0, decay rate reduces, and ringing continues for longer

Question 23
Record your findings.

At a_1 = +0.03, oscillations are self sustaining, without input.

Fosc = 2.75 x 0.2ms = 1.81kHz @ +/- 1.4Vpk ie: gains are -0.81, +0.03, 1.0

a1 = .03
σ = 0
a0 = .081
.9

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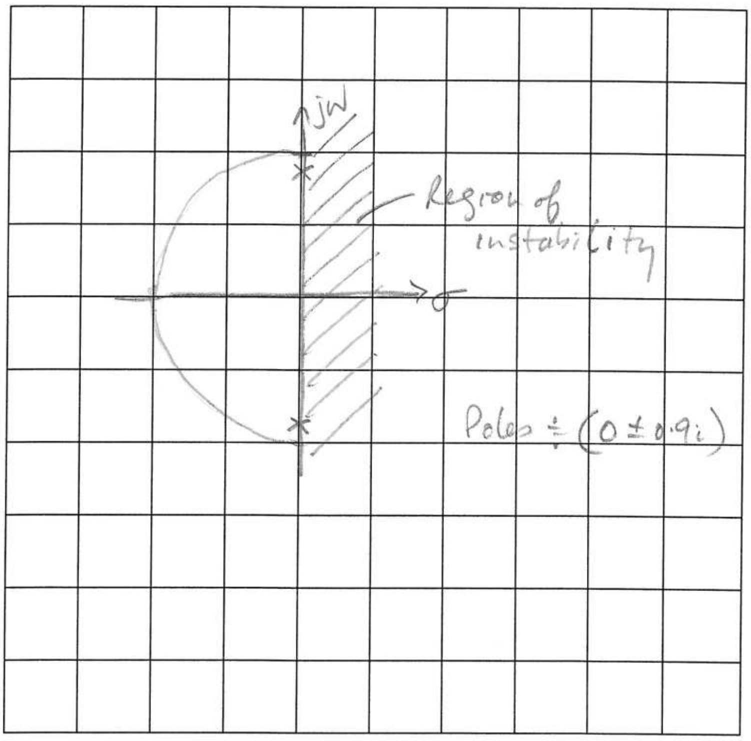


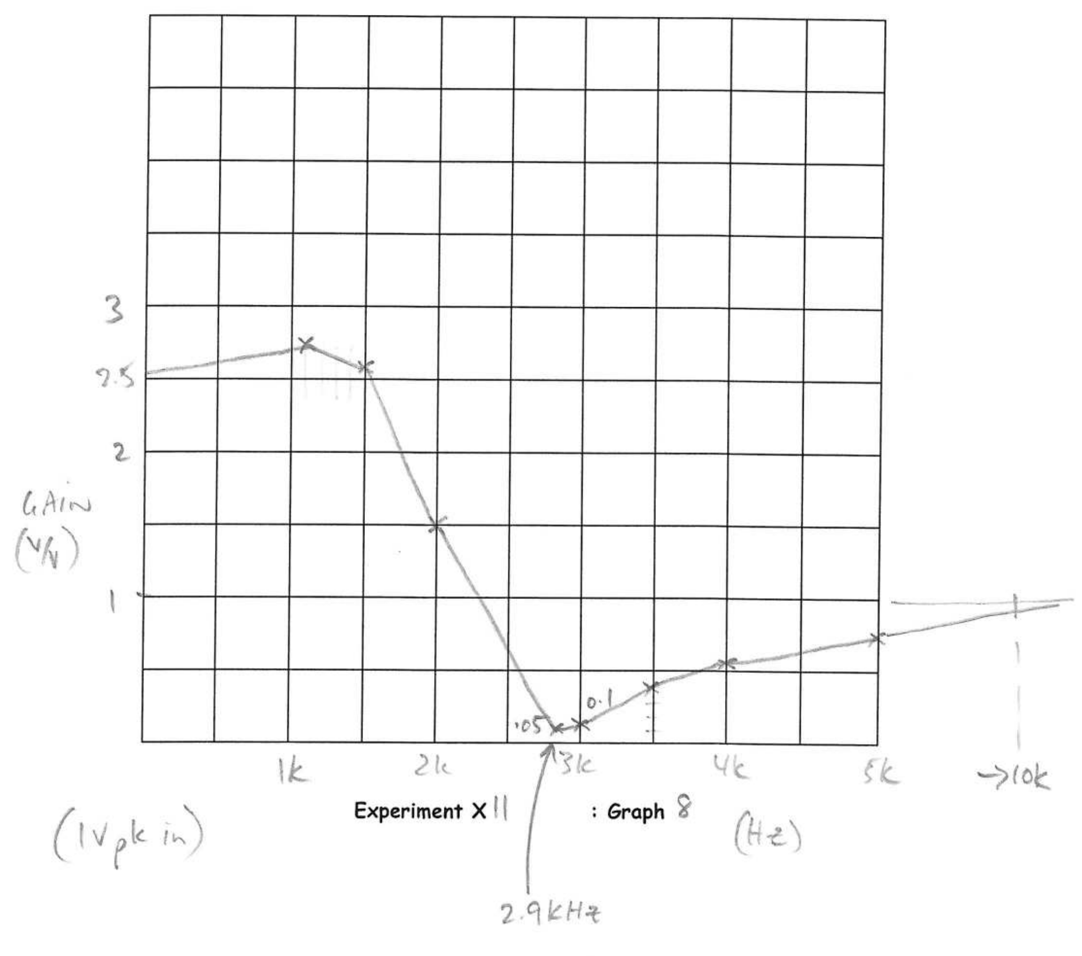
Experiment X 11 : Graph 7

Graph 3: pole locus with varying a1 coefficient

Question 24

Return to the setup in Fig. 3 and with a_0 back to the same position as in step 18, recorded in Q21, measure the resonance frequency at point x1 (the bandpass filter output). Compare this result with the time domain frequency measurements of the impulse response oscillations.

The frequencies are the same. The impulse reponse shows us the resonant frequency of the system.



Experiment X 11 : Graph 8

Graph 8: response

Question 25

Decrease b_0 progressively and observe that this causes a reduction of the gain at low frequencies. Continue until the gains at low and high frequencies are close to equal.
You may wish to use the manual GAIN ADJUST knob on the SIGEx board to vary this parameter. Remember to setup its range to suit your parameter.
Check that the null is still present. This realizes a bandstop filter, also known as a "notch" filter. Measure b_0 (and a_0 in case it was altered). Verify that b_2 is still set to unity.

Varying b0 causes gain reduction at low freq.

Equality occurs at b0=0.8. fnull = 1.8khz

Question 26

Show that this response is obtained when $b_0 = a_0$ (with $b_2 = 1$). This can be done quickly using Eqn 3 in prep Question 9: at low frequency, substitute $s = 0$; at high frequency use $1/s = 0$.

Refer to Q9

Question 27

From prep Question 11 we expect the deepest notch when b_1 is zero. Examine whether this is the case in your implementation. Vary b_1 above and below zero and find the value that gives the deepest notch. Suggest why there may be a discrepancy between theory and practice.
Check the integrator gain by comparing theoretical and measured values of the null frequency. Consider possible practical causes for any discrepancies.

b1=0 gives deepest notch

Theory & practice very close

Question 28

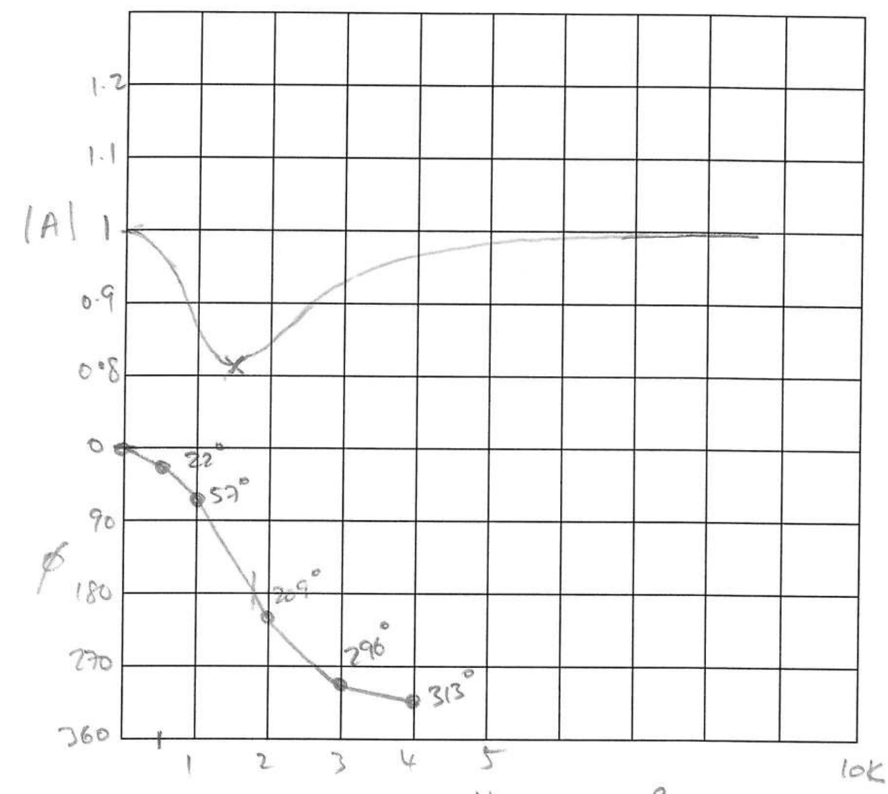
Select a different integrator constant: suggested dip switch position DOWN UP (i.e k around 29,000/s) and measure the new null frequency.

Fnull measured as 4.66kHz

Calculated new fnull by scaling of integration rate = 1.8k x (32000/12600) = 4.57k

Question 29

Measure and plot the phase shift vs frequency and, again compare with your expectations from the pole-zero plot, on Graph 9.



Experiment X 11 : Graph 9

1Vpk in

Graph 4:allpass responses

Question 30
Record the values of a_0 and a_1 that realize this outcome. This response is known as maximally flat. In Tut Q.8 you are invited to show that the formula for a maximally flat second order allpole is $a_1 = \sqrt{(2.a_0)}$.

a1=-1.3 looks satisfactory, though other similar values do as well.

Question 31
Record the values of a_0 and a_1 that realize this outcome. This response is known as critically damped. It is of interest in control systems as it realizes the most rapid risetime without overshoot. This idea also finds application in the context of Gaussian filters. Further exploration of critical damping is provided in Tut. Q.9.

A1= -1.54 gives a well damped response

Name: ______________________________________

Class: ______________________________________

X12 - Sampling and aliasing

SIGEX

Experiment 12 – Sampling and Aliasing

Pre-requisite work

Pre-lab preparation work

Question 1
Look up or derive the trigonometric identity for the product of two sines expressed as a sum. Confirm that the frequencies in this sum are (f1 + f2) and |f1 - f2|, where f1 and f2 are the input frequencies. Confirm that the output components are of equal magnitudes.

sin a. sin b = ½ (cos(a-b) – cos(a+b))

Question 2
Look up or derive the Fourier series of a squarewave (with no DC component) of duty ratio other than 50% (25% and 1% say). Note the sinx/x shaped spectrum envelope. Locate the frequency of the first null of the envelope for each case and note the relationship with the pulse width.

Now consider the 50% duty ratio case. Comment on the disappearance of the even harmonics.

50%: f(t)= 4/π[1.sin(wt) + 1/3.sin(3wt) + 1/5.sin(5wt) +…]for odd quarter-wave symmetry,A=1

Question 3
Derive the spectrum of the product of a sinewave and a 1% duty ratio squarewave. You can do this easily by using superposition with the results in Question 1 and Question 2. For convenience, make the frequency of the squarewave around five times the sinewave frequency. Plot the resulting spectrum.

Question 4
Repeat this for a few other sampling rates, from 2000Hz, down to 400Hz, say. Document your readings in Table 1 below. From these observations, what is the minimum sampling rate you consider adequate to allow recovery of the analog signal without too much distortion, on the basis of this sampling format (i.e. using the SAMPLE/HOLD function).

Recovery is getting sensitive and difficult to achieve around 400 Hz

Sample rate (Hz)	TLPF setting (approx.position)	Recovered amplitude (V)
2000	9 o'clk	1.7V
1000	8.30 o'clk	1.7V

800	8:15 o'clk	1.7V
400	8 o'clk	1.7V

Table 1: sample rate readings for recovery from S/H

Question 5
Repeat the procedures in step 15 for recovery using the TUNEABLE LPF using the sample train generated with the system in Fig 2, i.e. with narrow pulses. Document your readings in Table 1 below. Compare the outcome with those obtained with the S/Hold method. Do you expect one of these sample formats to be better for interpolation to analog form? Is this borne out by your results?

S/H should be better, due to less transitions

No. Isolating the fundamental is the only issue.

Table 2: sample rate readings for sampled pulse train recovery

Sample rate (Hz)	TLPF setting (approx.position)	Recovered amplitude (V)
2000	9 o'clk	1V
1000	8.30 o'clk	1V
800	8:15 o'clk	1V
400	8 o'clk	1V

Question 6
Examine the step and impulse responses of the filter at the settings that give you the best outcomes. Measure risetime and related properties and compare with the sample interval. [1] Use the PULSE GENERATOR module set to 10Hz, and various DUTY CYCLES settings to achieve this easily.

Risetime=5ms; ringing @ 100Hz

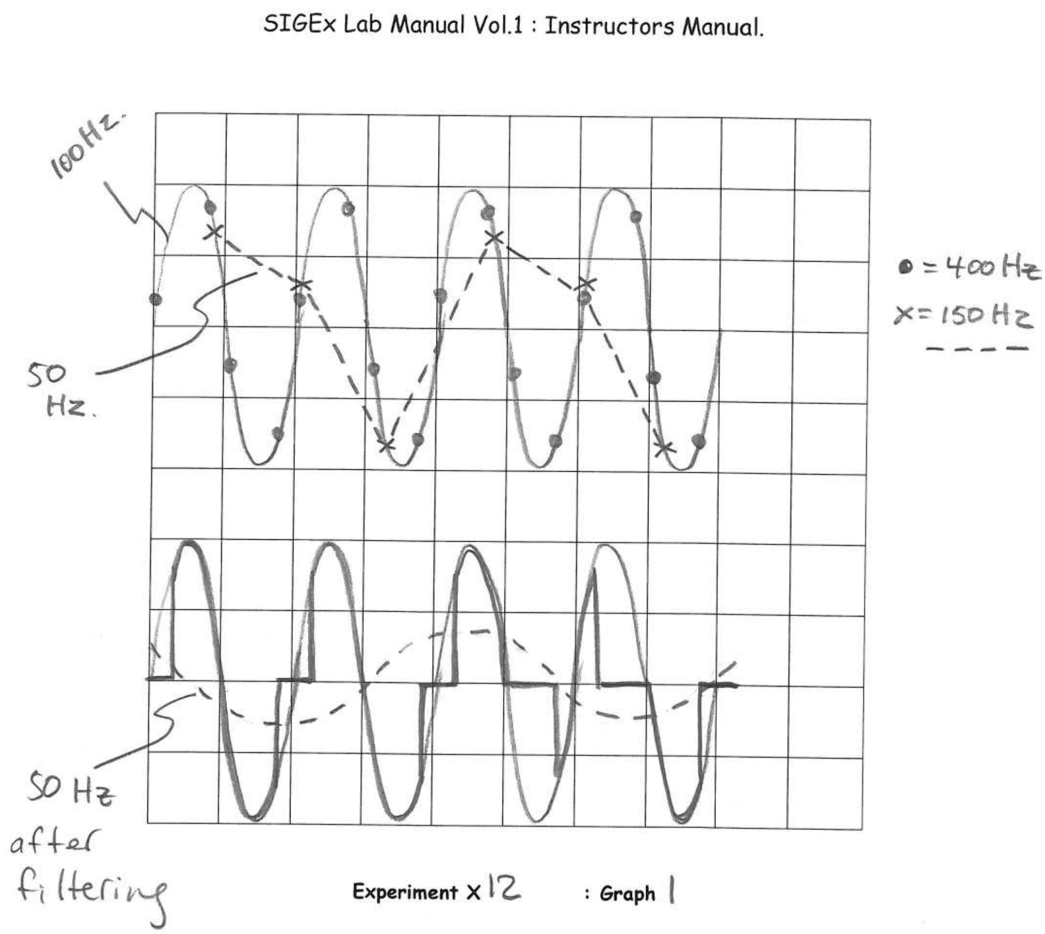
Width of impulse = 10ms

Question 7
For the same settings as in step 17, carry out a quick examination of the frequency response of the filter. Obtain and record the 3dB cut-off frequency, and the attenuation of the stop-band.

DC=3.5, hence -3db = 2.47V

[1] You may wish o refer back to your notes from "Experiment 3: Special signals", where step and impulse responses were covered.

f-3db=110Hz. Stopband attenuation @ 200Hz > -60dB



Graph 1: alias waveforms

Question 8
Explain why the sampled signal spectrum looks the way it does and specifically relate this to your understanding of pre-lab preparation item 1 & 2.

By superposition and sums/differences, each sampling signal harmonic has upper & lower sidebands of the sampled signal.

Question 9
Note the frequency of the first and second nulls in the spectrum and explain why they are at those frequencies.

Fnull = n * 1/pulse width = n * 4kHz

Question 10
At what sampling rate does the lower sideband of the first spectrum image become located at the same frequency as the input sinewave ?

200

Question 11
You should be able to recover a clean sinewave. What is its frequency ? Where does it come from?

50 Hz. It is a created "alias" or "image" component

Question 12
Why is it not possible to recover the analog input when the number of samples per cycle of the input sinewave is less than two?

Less than 2 samples per cycle causes false frequency components to be created.

Question 13
What is the minimum sampling rate that allows a filter to be able to recover the original sinewave signal without any other unwanted components ?

Slightly more than two times the signal frequency, due to filter not being a perfect "brickwall" as required by theory.

Name: ____________________________________

Class: ____________________________________

X13 - Getting started with analog-digital conversion

Experiment 13 – Getting started with analog-digital conversion

Question 1
Show that n = $\log_2(L)$:

An 'n' bit frame represents 2^n (= L) possible states, hence n = $\log_2 L$

Question 2
Record the number of clock periods per frame.

8

Question 3
Currently the analog input signal is zero volts (since INPUT is grounded). Before checking with the scope, consider what the PCM encoded output might look like. Can you assume that it will be 00000000?. What else might it be, bearing in mind that this PCM ENCODER outputs *offset binary* format.

10000000 = 0V

Question 4
On CH1 display the signal at PCM DATA output.The display should be similar to that in Figure 3 (possibly with fewer frames). Is it in agreement with your expectations?

FS:00000001

DATA: 01111110, out by 2 bits. Reason: PCM encoder is not calibrated to 0V

Question 5
Adjust VARIABLE DC to its maximum negative value. Record the DC voltage and the pattern of the 8-bit binary number.

-2.5V = 00000000

Question 6
Slowly increase the amplitude of the DC input signal until there is a sudden change to the PCM output signal format. Record the format of the new digital word, and the input DC voltage at which the change occurred. Use the INCREMENT arrows on the digital value entry box for a steady stable increase in DC value.

-2.44 = 00000001

Table 1: DC VOLTAGE input vs. PCM codewords

DC VOLTAGE (V)	8 bit PCM codeword
-2.5	00000000
-2.44	00000001
-1.54	00101111
-0.58	01100000
+0.16	10000110
+0.89	10101011
+1.75	11010111
+2.49	11111101
+2.5	11111110

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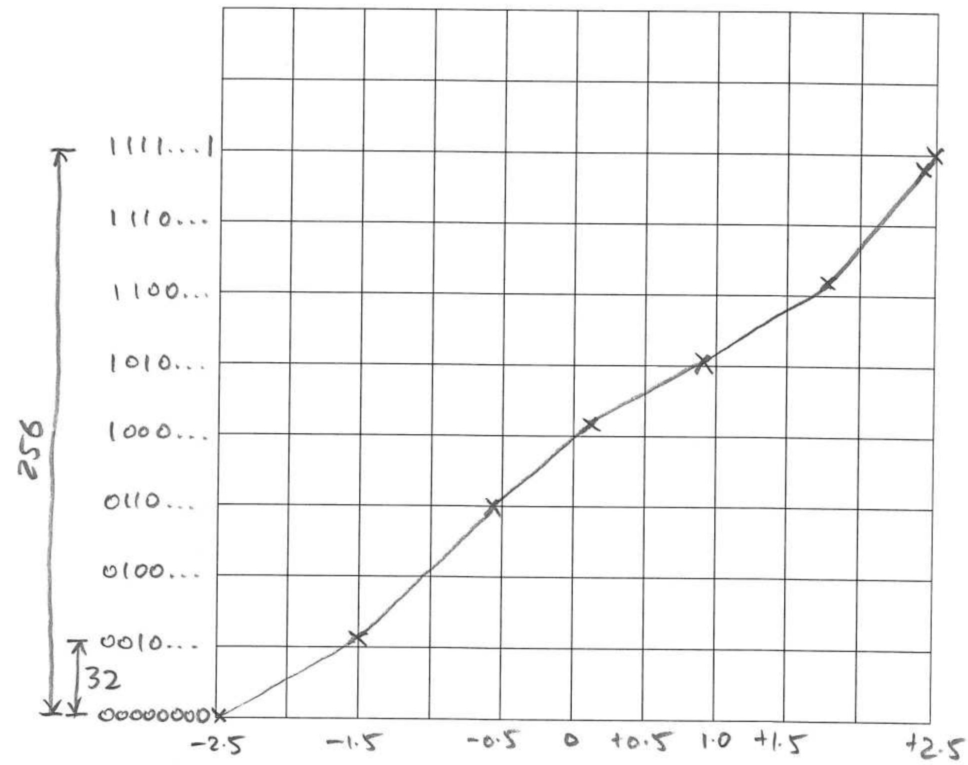


Experiment X 13 : Graph 1

Graph 1:DC to binary word plot

Question 7

On the basis of your observations so far, provide answers to the following:

* what is the sampling rate ?
* what is the frame width ?
* what is the width of a data bit ?
* what is the width of a data word ?
* how many quantizing levels are there ?
* are the quantizing levels uniformly (linearly) spaced ?
* what is the the minimum quantized level spacing ? How does this compare to theory ?

Fs=10k/8 = 1.25ksamples/sec

8 bit; Tbit = 1/10,000 sec; Tword = 1/10,000*8 sec; 256 levels; Yes

Measured quantizing levels: 2.5-2.32 for 7 levels = 0.18V/7 = 0.24V/level

Theory: 5V/256 = 0.02V/level

Question 8

The relationship between the sampled input voltage and the output codeword has been described above. Suggest some variations of this relationship that could be useful ?

Compressing certain regions of the scale can be useful to increase/decrease resolution

in those regions eg: companding.

Question 9

Adjust the scope to display this waveform. Record its shape and frequency. Check whether this conforms with the Nyquist criterion. Show your reasoning.

100Hz sine, 2V pk. Min sampling rate = 200Hz

Fsampling = 10k/8=1.25kHz >> 200Hz

Question 10

Momentarily, vary the clock rate from 10,000 to 20,000 Hz. How does this affect the "sampling distortion" viewable in the output signal ?

The quantization reduces.

Question 11

View the input to the TUNEABLE LPF, ie the output of the PCM DECODER and compare with the INPUT sinusoid. What is the gain of the PCM DECODER itself.

Approx 6/8

Question 12
Can you explain the source of the delay between input and output signals ? Both with and without the TUNEABLE LPF ?

PCM data frame transmission time, PCM data frame reception time, and analog filter delay.

Question 13
Momentarily, vary the clock rate from 10,000 to 20,000 Hz. How does this affect the required Fc needed to recover the signal without distortion ?

Higher Fc is adequate for the 20k case, due to the sampling images being further apart.

Name: ____________________________________

Class: ____________________________________

X14 - Discrete-time filters with FIR systems

SIGEX

Experiment 14 – Discrete-time structures:

Preparation

This preparation provides essential theory needed for the lab work to make sense.

Question 1
Consider the system in Figure 1, where nT are the discrete-time points, with T sec denoting the unit time delay, i.e. the time between samples. Show that the difference equation relating the output y(nT) and the input u(nT) is

$$Y(nT) = b_0.u[nT] + b_1.u[(n-1)T] + b_2.u[(n-2)T] \qquad \text{(Eqn 1).}$$

Show by substitution that $e^{jnT\omega}$ is a solution, i.e. show that when the input is $e^{jnT\omega}$, y(nT) is $e^{jnT\omega}$ multiplied by a constant (complex-valued); ω is the frequency of the input in radians/sec.

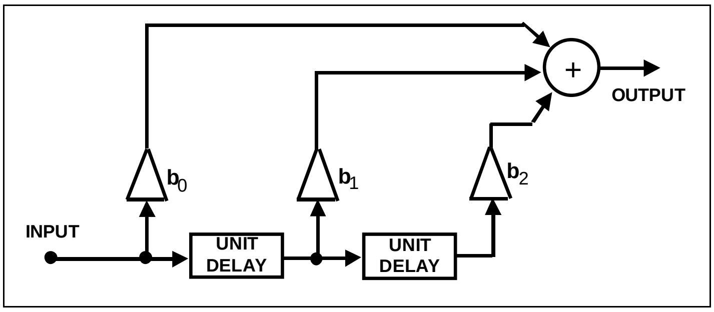


Figure 1: schematic of FIR filter with two unit delays

In Lab 11 we used a complex exponential input to represent the behaviour of a system that is supposed to operate with real-valued signals. You could consider using u[nT] = cos(nTω) or sin(nTω) instead. However, the use of the exponential function simplifies the math considerably. We have already seen that cos(ωt) is Re{exp(jωt)}, so, you can carry out the analysis with $e^{jnT\omega}$, then simply take the real part of the result. After a while, working with complex exponential functions to represent sinusoids becomes second nature and we don't even bother thinking about taking the real part. Many practical systems implemented digitally actually operate with complex-valued signals, for example modulators and demodulators working with quadrature signals.

From the above, with input $u(nT) = e^{jnT\omega}$, show that

$$H = y/u = b_0 + b_1.\ e^{-jT\omega} + b_2.\ e^{-j2T\omega} \qquad \text{(Eqn 2).}$$

Note that H is not a function of n.

Question 2
Use this result to obtain a general expression for the magnitude of y/u as a function of ω. You will need to first write down the real and imaginary parts.

Pre-lab preparation work

Set T = 1 sec for the time being, and plot the result for the case $b_0 = 1$, $b_1 = -1.3$, $b_2 = 0.9025$ over the range ω = 0 to 2.π rad/sec. Label the frequency axis in Hz as well as rad/sec. You should find there is a significant dip in the response near 0.13Hz.

Question 3

As in Lab 7, we consider an alternative way of getting frequency responses. We will create a graphical medium to provide an intuitive environment for visualizing and generating both magnitude and phase responses.

First, return to the expression for y/u obtained in (a) and replace "exp(jTω)" by the symbol "z". Look upon z merely as a convenient macro for exp(jTω) . At this point there is no need to ascribe any deeper significance to this substitution. The result is the (complex-valued) polynomial

$$y/u = H(z) = b_0 + b_1.z^{-1} + b_2.z^{-2} = z^{-2} . [b_0. z^2 + b_1.z + b_2] \qquad \text{(Eqn 3).}$$

For the case $b_0 = 1$, $b_1 = - 1.3$, $b_2 = 0.9025$ (from (Q2)), express the quadratic in the brackets in the factored form $(z - z_1)(z - z_2)$, where z_1 and z_2 are the roots. Show that these are given by

$$z_1 = 0.95e^{\,j0.260\pi}$$
$$z_2 = 0.95e^{\,-j0.260\pi} \qquad \text{(Eqn 4).}$$

Satisfy yourself that the magnitude response of H can be expressed as

$$|H(\omega)| = |(e^{jT\omega} - z_1)|.|(e^{jT\omega} - z_2)| \qquad \text{(Eqn 5).}$$

Write down the corresponding expression for the phase of H.

Question 4: Graphical plotting of poles & zeros

We are now ready to proceed with a graphical approach for evaluating the factors $(z - z_1)$ and $(z - z_2)$ in Eqn 3. Place an "o" on a complex plane (we will refer to this as the z plane) at the locations corresponding to z_1 and z_2, as obtained in Eqn 4. With T = 1, we will get an estimate of |H| at ω = π/5.

Place a dot at the point $e^{j\pi/6.}$ Join this point and the point z1 with a straight line. The length of this line is $|(e^{j\pi/5}- z_1)|$.
Do the same with z_2 to obtain $|(e^{j\pi/5} - z_2)|$. <u>From Eqn 5, the desired estimate of |H(π/5)| is simply the product of the lengths of these two lines.</u>

Question 5

By repeating this for other values of ω we are able to get a quick estimate of the graph of |H| versus ω. It's important to note that the locus of $e^{jT\omega}$ is a circle of unity radius centered at the origin (known as the *unit circle*). Hence, the general shape of the frequency response is easily estimated by simply running a point counter-clockwise along the circumference of the unit circle, starting at (1, 0). Note that the idea is just a variant on the procedure introduced in Lab 11, where we moved the frequency point along the j axis. Compare the outcome with the result computed in (Q2).

Experime

Notice that the presence of the trough in the response can be seen at a glance from the behaviour of the vector from the "zero" z_1 as the dot on the unit circle is moved near z_1. By comparison, the rate of change of the other vector is small over that range.

Question 6

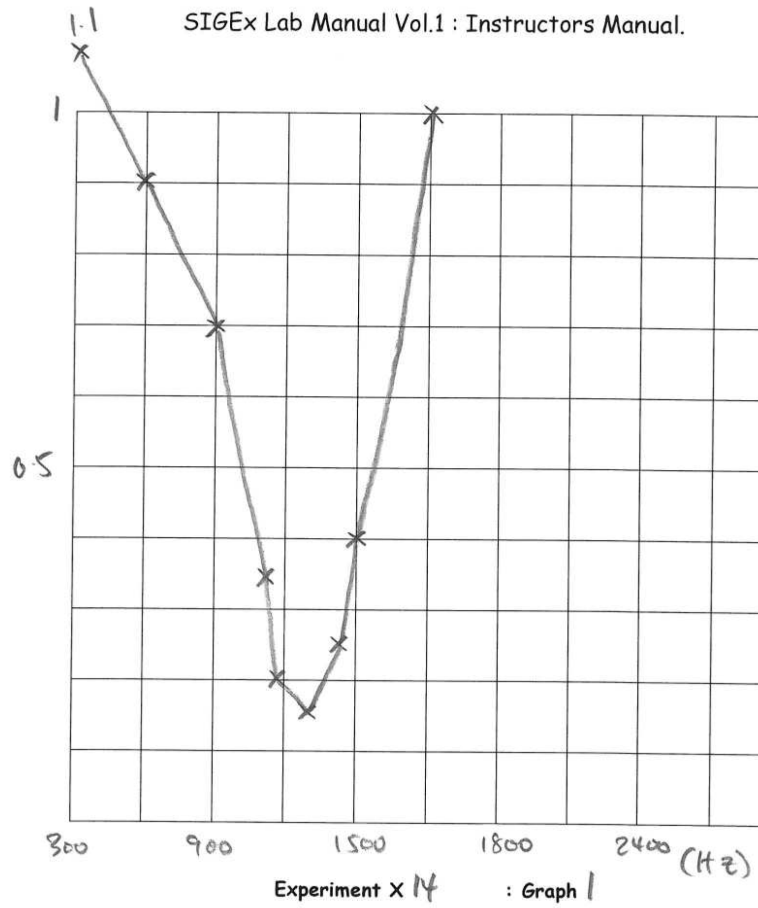
Modify Fig 1 by replacing the unit delays with a gain of 1/z and show that Eqn 3 follows by inspection using simple algebra, without the need to work through the difference equation step. While this is only a minor simplification in this example, it is very useful in more complicated cases, especially where feedback loops are involved.

Although z was originally introduced in (Q3) as just a substitution for $e^{jT\omega}$, our interpretation appears to have been extended in (Q4) to include any complex number. Consider whether this is the case, and why.

Question 7

In the above example, we had the sample interval T = 1. Suppose T = 125 microsec. Adjust the frequency axis for this value of T. Extend this result for any value of T. How are the zeros of H(z) affected by the value of T? Why is it appropriate to use T = 1 normally?

Show that $|H(\omega)|$ is periodic, and determine the period in Hz.



Graph 1:response plot

Question 8

Measure the notch frequency and the depth relative to the response at DC. Also measure the time delay as a function of frequency at several points of interest.

Fn = 1.26kHz, attenuation = 0.15/1.1

Question 9

Determine and note the new notch frequency, for the b_1 gain entered. Document the relationship between b_1 and notch frequency

Question 10

What is the level of attenuation of the f2 signal for the original zero positions.

F1: 2V in, 0.2V out pk

F2: 2V in, 1V out pk … using FFT display

Question 11

From your previous findings in this experiment, what change is required to gain b1 to reduce the notch frequency ?

As b1 is reduced toward – 2, fnotch reduces

Question 12

What is the equation relating theta of the zero to the frequency of the zero, as implemented in the PZ PLOT TAB ?

Zero frequency = zero theta(deg)/360 x clock frequency.

Question 13

For what value of b1 did you achieve the maximum attenuation of the lower message component RELATIVE to the higher component ? What levels did you measure ?

B1=-1.81

F1=0.1V; f2=1V

Question 14

What components is the TUNEABLE LPF attenuationg in order to give a "clean" signal ?

It is predominantly eliminating the image harmonics around 10kHz, the sampling clock rate.

It is these harmonics which create the sampled/stepped nature of the discrete output signal.

Name: ______________________________

Class: ______________________________

X15 - Poles and zeros in the z-plane with IIR systems

SIGEX

Experiment 15 – Poles and zeros in the z plane: IIR systems

Question 1
Consider the feedback system in Figure 1.

Show that the difference equation relating the adder output x0(nT) and the input u(nT) is

$$x_0(nT) = u[nT] - a_1.x_0[(n-1)T] - a_2.x_0[(n-2)T] \qquad \text{(Eqn 1)},$$

where nT are the discrete time points, T sec denoting the unit delay, i.e. the time between samples.

Show by substitution that e^{jnTw} is a solution, i.e. show that when $x_0(nT)$ is of the form e^{jnTw}, the input u(nT) is e^{jnTw}, multiplied by a constant (complex-valued); w is the frequency of the input in radians/sec; (the use of complex exponentials for the representation of sinusoidal signals is discussed in Lab 8, 10 and 13.

From the above, with input $u(nT) = e^{jnTw}$ obtain

$$x_0/u = 1/[\,1 + a_1.\,e^{-jTw} + a_2.\,e^{-j2Tw}] \qquad \text{(Eqn 2)}.$$

Note that x_0/u is not a function of the time index n.

SIGEx Expt 15 Poles and Zeros in z plane (IIR)
Prep Solutions

Q1 From the block diagram, at time nT, the sum at the adder output is

$$x_0(nT) = u(nT) - a_1 x_1(nT) - a_2 x_2(nT) \quad \text{(Eqn 1.1)}$$

Along the delay line we have

$$x_2(nT) = x_1((n-1)T)$$
and $$x_1(nT) = x_0((n-1)T)$$
hence $$x_2(nT) = x_0((n-2)T)$$

Substituting into Eqn 1.1, we obtain the required result:

$$x_0(nT) = u(nT) - a_1 x_0((n-1)T) - a_2 x_0((n-2)T)$$

Next, we show that $x_0 = A\exp(jnT\omega)$ is a solution when the input $u(nT) = \exp(jnT\omega)$. First, move all the x_0 terms to the LHS of the difference eqn:

$$x_0(nT) + a_1 x_0((n-1)T) + a_2 x_0((n-2)T) = u(nT).$$

Substitute $A\exp(jnT\omega)$ for x_0 in LHS:

$$LHS = A\exp(jnT\omega) + a_1 A\exp(j(n-1)T\omega) + a_2 A\exp(j(n-2)T\omega)$$
$$= A\exp(jnT\omega)\left[1 + a_1\exp(-jT\omega) + a_2\exp(-j2T\omega)\right]$$

For given values of a_1, a_2 and $T\omega$ the quantity in [ ] is a constant (i.e. not a function of the time index n.)
If we select the value of A such that

$$A.\left[1 + a_1\exp(-jT\omega) + a_2\exp(-j2T\omega)\right] = 1$$

then the $RHS = \exp(jnT\omega) = LHS$
i.e. $x_0 = A\exp(jnT\omega)$ is a solution QED.

From the above we have $\dfrac{x_0}{u} = \dfrac{A\exp(jnT\omega)}{\exp(jnT\omega)} = A$

with $A = \dfrac{1}{1 + a_1\exp(-jT\omega) + a_2\exp(-j2T\omega)}$

END Q1

Question 2
Use this result to obtain a general expression for $|x_0/u|$ as a function of w.

Tip: to simplify the math, operate on u/x_0 instead of x_0/u, expressing the result in polar notation.

Set T = 1 sec for the time being, and plot the result for the case $a_1 = -1.6$, $a_2 = 0.902$ over the range w = 0 to π rad/sec. Label the frequency axis in Hz and in rad/sec. You should find there is a peak in the response near 0.09Hz.

SIGEx Expt 15 Poles and zeros in z plane
Prep solutions

Q2 General expression for $|A|$:

$$\frac{1}{|A|} = \left| 1 + a_1 \exp(-jT\omega) + a_2 \exp(-j2T\omega) \right|$$

$$= \left| 1 + a_1 \left(\cos(T\omega) - j\sin(T\omega)\right) + a_2\left(\cos(2T\omega) - j\sin(2T\omega)\right) \right|$$

$$= \left| \left(1 + a_1 \cos(T\omega) + a_2 \cos(2T\omega)\right) - j\left(a_1 \sin T\omega + a_2 \sin 2T\omega\right) \right|$$

$$= \sqrt{\left[\left(1 + a_1 \cos T\omega + a_2 \cos 2T\omega\right)^2 + \left(a_1 \sin T\omega + a_2 \sin 2T\omega\right)^2 \right]}$$

An alternative expression can be obtained using

$$|A|^2 = AA^* \quad \text{where } A^* \text{ is the conjugate.}$$

END Q2

Q3 The roots of the quadratic are $-\frac{a_1}{2} \pm \sqrt{\left(\left(\frac{a_1}{2}\right)^2 - a_2\right)}$

Consider the case $\left(\frac{a_1}{2}\right)^2 < a_2 \Rightarrow$ complex valued roots

$$-\frac{a_1}{2} \pm j\sqrt{\left(a_2 - \left(\frac{a_1}{2}\right)^2\right)} \doteq p_1 \,,\ p_1^*$$

Now convert from Cartesian to polar form:

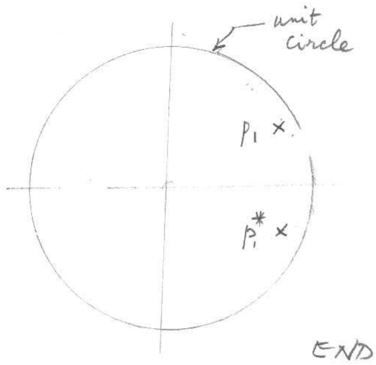
$$|p_1|^2 = \frac{a_1^2}{4} + a_2 - \frac{a_1^2}{4} = a_2 \Rightarrow |p_1| = \sqrt{a_2}$$

$$\measuredangle p_1 = \text{inv}\cos\left(\frac{-a_1/2}{\sqrt{a_2}}\right) \ \text{(radians)}$$

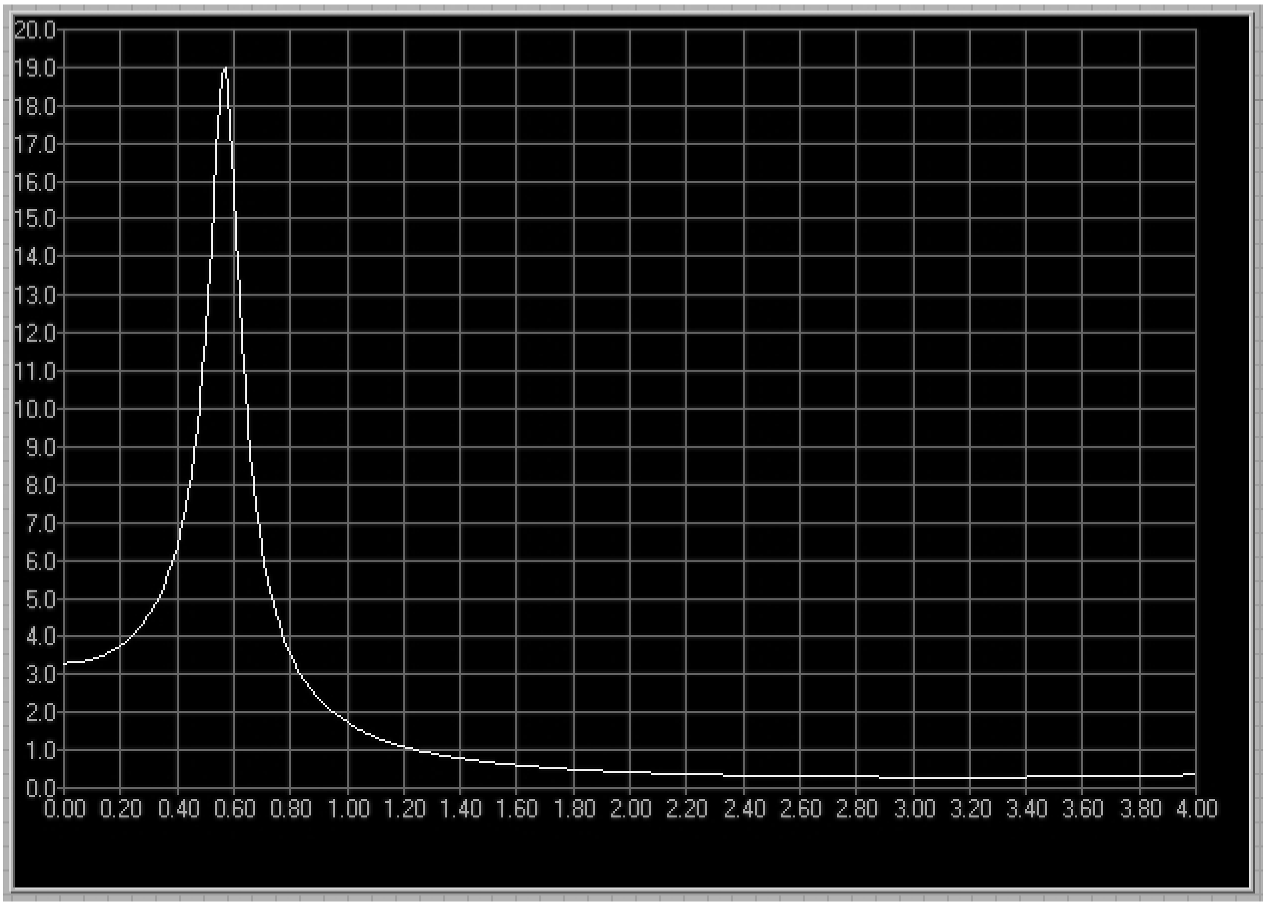
For $a_1 = -1.6$, $a_2 = 0.902$

$p_1 = 0.95 \exp(j\, 0.570)$

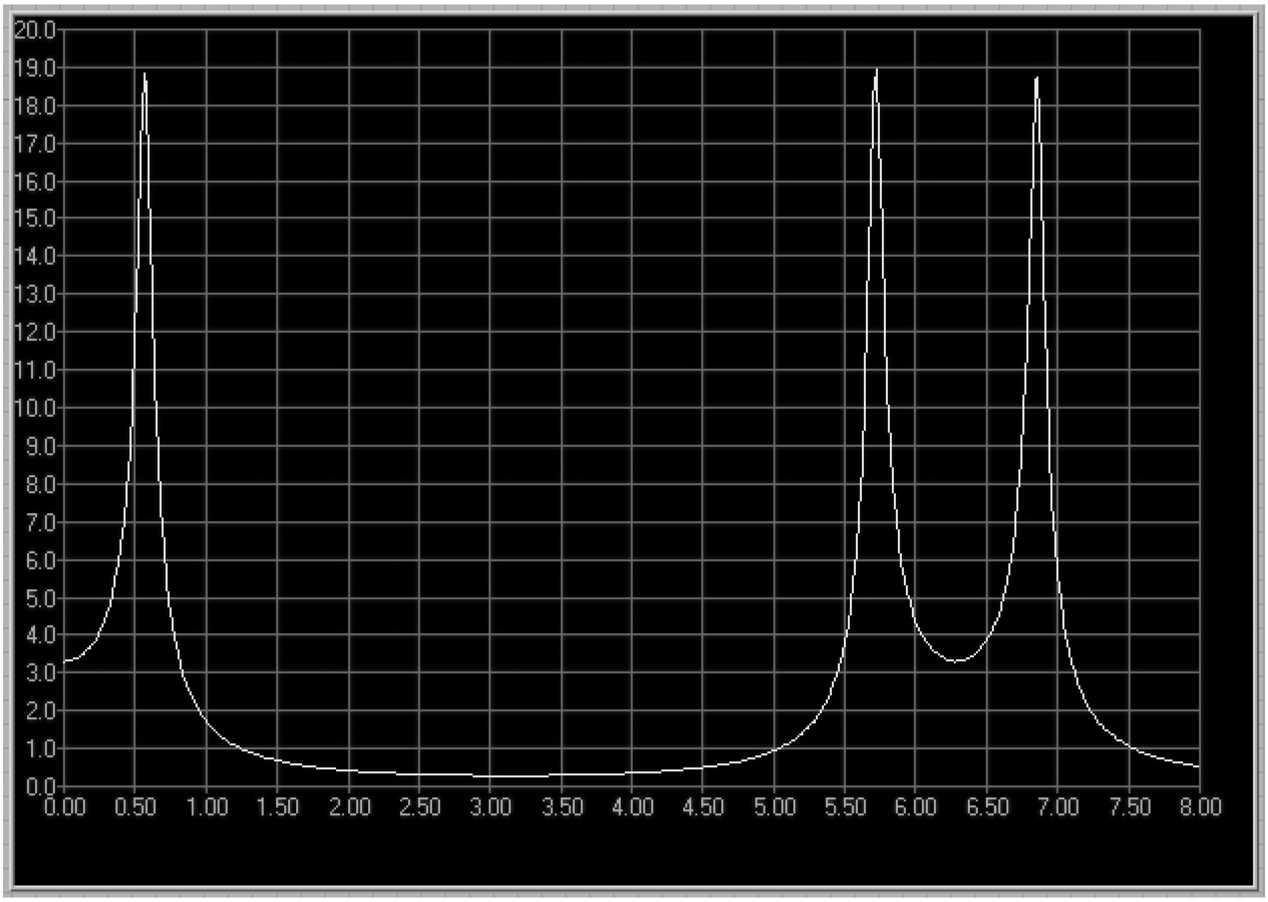
$p_2 = 0.95 \exp(-j 0.570)$



END Q3



F(w) = 1/sqrt[(1+acos(w)+bcos(2w))^2 + (asin(w)+bsin(2w))^2)] for a=-1.6, b=0.9025



F(w)=abs[1/bexp(i2w)+aexp(iw) + 1] for a=-1.6, b=0.9025

Question 3

Replace "exp(jTw)" by the symbol "z" in Eqn 2. The result is

$$H\_x_0(z) = x_0/u = 1/(1 + a_1.z^{-1} + a_2.z^{-2}) = z^2 / (z^2 + a_1.z + a_2) \qquad \text{(Eqn 3).}$$

The quadratic ($z^2 + a_1.z + a_2$) can be expressed in the factored form $(z - p_1)(z - p_2)$.
Using the values of a_1 and a_2 given in Question 2 above, find the roots p_1 and p_2 (express the result in polar notation). Mark the position of p_1 and p_2 on the complex z plane with an "x" to indicate that they represent poles. The distance between these points and the unit circle is of key importance.

This is a parallel process to that in Lab 11 where we plotted zeros. A similar procedure was carried out in Lab 11 for a CT transfer function in the complex variable s.

Write down a formula for p_1 in terms of a_1 and a_2. Note that p_1 may be real or complex depending on a_1 and a_2. Determine the conditions for p_1 to be complex valued. For this case, express p_1 in polar notation. Take note of the fact that $|p_1|$ does not depend on a_1 (this will be useful later). Obtain p_2 from p_1.

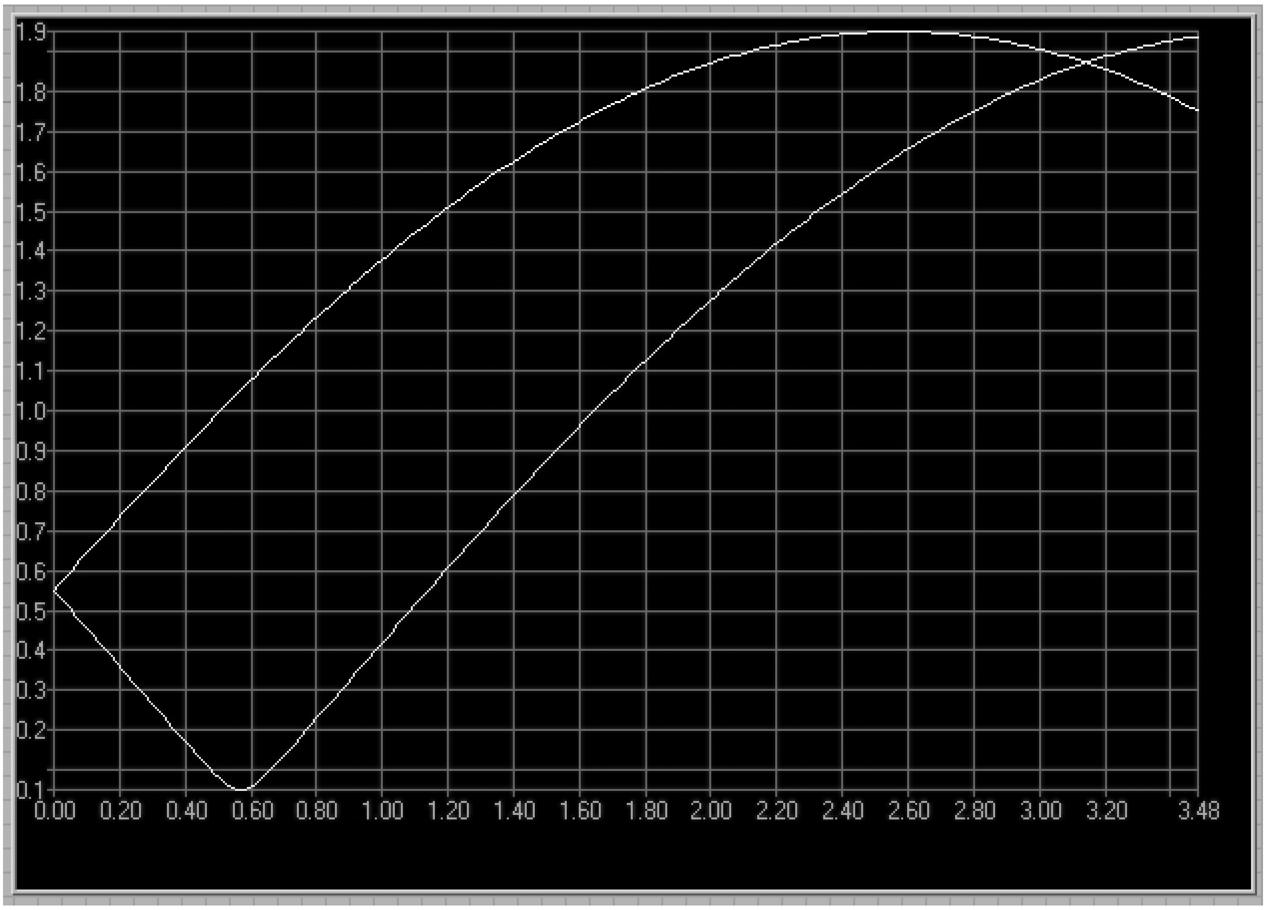
Question 4

Satisfy yourself that the magnitude response of H_x_0 is given by

$$|H\_x_0(w)| = 1/[|(e^{jTw} - p_1)|.|(exp^{jTw} - p_2)|] \qquad \text{(Eqn 4).}$$

This provides the key for the graphical method described in Lab 13 to obtain an estimate of the magnitude response. Again, we will use T = 1 .

Plot the magnitude of the denominator for selected values of w over the range 0 to π. The quantity $|(e^{jTw} - p_1)|$ becomes quite small and changes rapidly as the point on the unit circle is moved near p_1. Plot additional points there as needed. Invert to get $|H\_x_0(w)|$ and compare this with the result you obtained in (b).

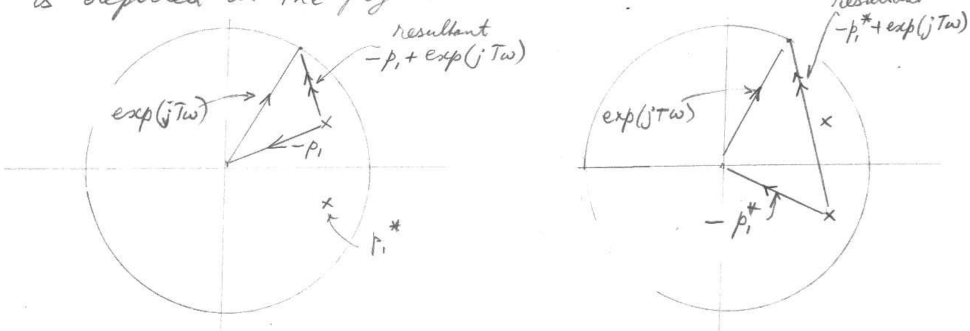


Q4: F(w) = abs[exp(iw) – 0.95exp(+/- i0.56962]

SIGEx Expt15 p-z in the z plane (IIR)
prep solutions

Q4 From Q3, $(\exp(jT\omega)-p_1)(\exp(jT\omega)-p_1^*)$ is the factored form of the quadratic in the denominator in Eqn 2; i.e. Eqn 4 is an alternative representation of Eqn 2.

The magnitude of this quadratic can be estimated graphically as the product of the lengths of the complex plane phasors $(\exp(jT\omega)-p_1)$ and $(\exp(jT\omega)-p_1^*)$. The representation of these phasors is depicted in the figure below.



Phasor diagram for p_1 factor Phasor diagram for p_1^* factor

The phasor $\exp(jT\omega)$ traces a circle of unity radius, known as the unit circle. The angular position of $\exp(jT\omega)$ represents the normalized frequency over the range 0 to π, i.e. 0 to Nyquist.
It can be seen from the phasor diagrams that each factor of the quadratic is the sum of $-p_1$ and $\exp(jT\omega)$, i.e. a phasor originating at the pole position and terminating at the point $\exp(jT\omega)$.
For the coefficients in Q2 $p_1 = 0.95\exp(j0.57\overline{0})$.
Graphs of the magnitude of each factor of the quadratic and of the product as a function of normalized frequency are shown in Fig xxx.
From these graphs, over the frequency range of principal interest, i.e 0 to π, we see that the contribution of the pole in the upper half plane is dominant, especially in proximity of the pole. ctd →

SIGEx Expt 15 p-z in the z plane (IIR)
prep solutions

Q4 ctd.

When $\exp(jT\omega)$ is near a pole the rate of change of the corresponding contribution is much greater than that of the contribution of more distant poles, hence its dominance in the product. All of the above applies also to factors in the numerator, associated with zeros of the transfer function.

END Q4.

Q5 System equation using z^{-1} to represent a unit delay:

The output of the adder $x_0 = u - a_1 x_1 - a_2 x_2$

$$x_1 = \frac{x_0}{z} . \quad x_2 = \frac{x_0}{z^2}$$

$$\Rightarrow \quad x_0 + a_1 z^{-1} x_0 + a_2 z^{-2} x_0 = u$$

$$\Rightarrow \quad x_0 (1 + a_1 z^{-1} + a_2 z^{-2}) = u$$

$$\Rightarrow \quad \frac{x_0}{u} = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

QED.

Q6 From Fig 3

$$y = b_0 x_0 + b_1 x_1 + b_2 x_2$$

$$= b_0 x_0 + b_1 z^{-1} x_0 + b_2 z^{-2} x_0$$

$$= x_0 (b_0 + b_1 z^{-1} + b_2 z^{-2})$$

$$\Rightarrow \frac{y}{u} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

QED

END Q6

Question 5
Modify Fig 1 by replacing the unit delays with a gain of 1/z and show that Eqn 3 follows by inspection using simple algebra.

Question 6
Apply this idea to show that the transfer function for the system in Fig. 3 is

$$H\_y(z) = y/u = (b_0 + b_1.z^{-1} + b_2.z^{-2}) / (1 + a_1.z^{-1} + a_2.z^{-2}) \qquad \text{(Eqn5)}$$

Question 7
Use the graphical pole-zero method (covered in Experiment 14) to obtain estimates of the magnitude responses for the following cases (0 to Nyquist freq):

(i) $b_0 = b_2 = 1$, $b_1 = 2$, a_1 and a_2 as in Question 2.
(ii) $b_0 = b_2 = 1$, $b_1 = -2$, a_1 and a_2 as in Question 2
(iii) $b_0 = 1$, $b_1 = 0$, $b_2 = -1$, a_1 and a_2 as in Question 2

Which of these is lowpass, highpass, bandpass?

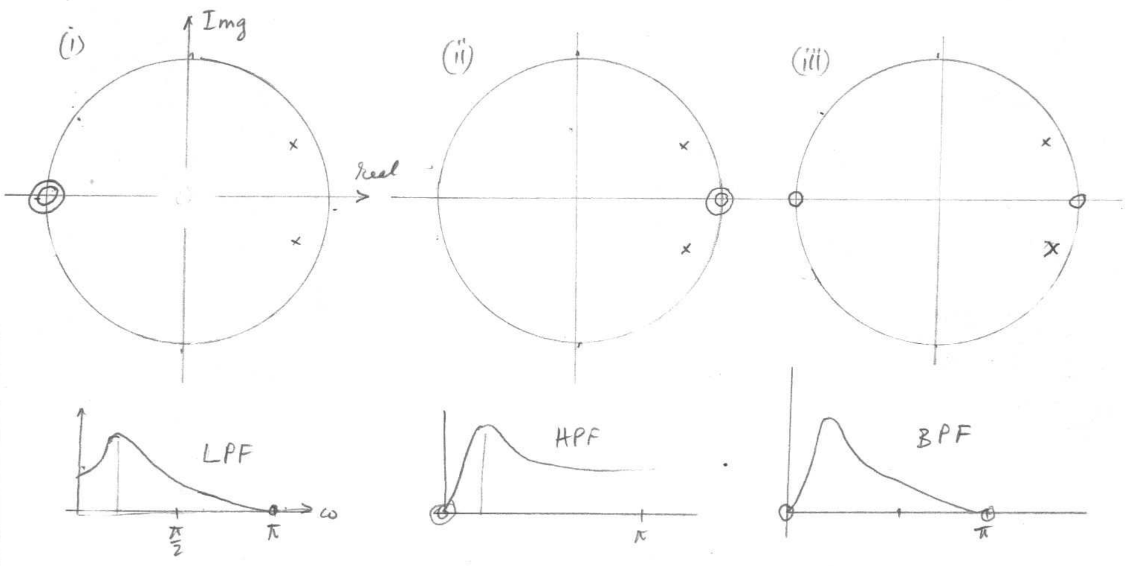
SIGEx15 p-z in zplane (IIR)
Prep Solutions

Q.7 We use $H_y(z) = \dfrac{b_0 z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2}$

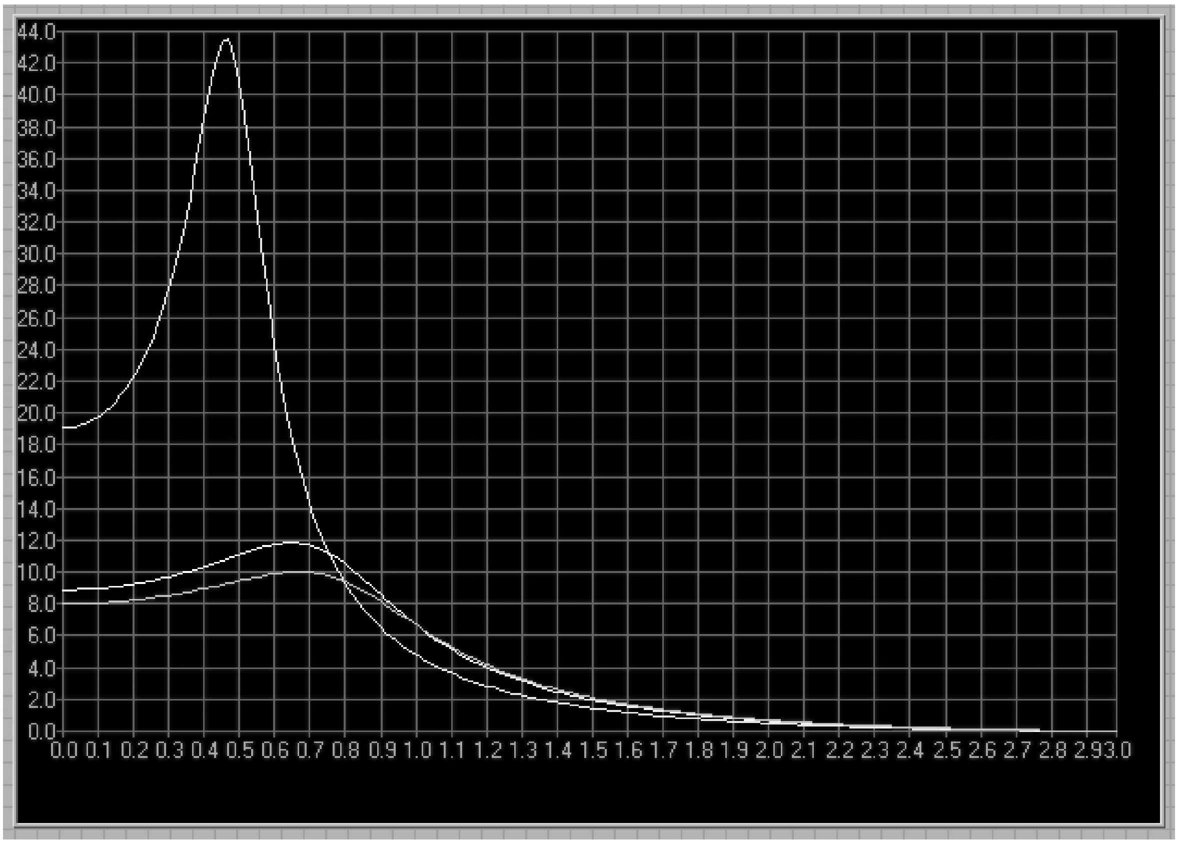
Case (i) numerator $= z^2 + 2z + 1 = (z+1)^2$ => two zeros at $z = -1$

Case (ii) numerator $= (z-1)^2$ => two zeros at $z = 1$

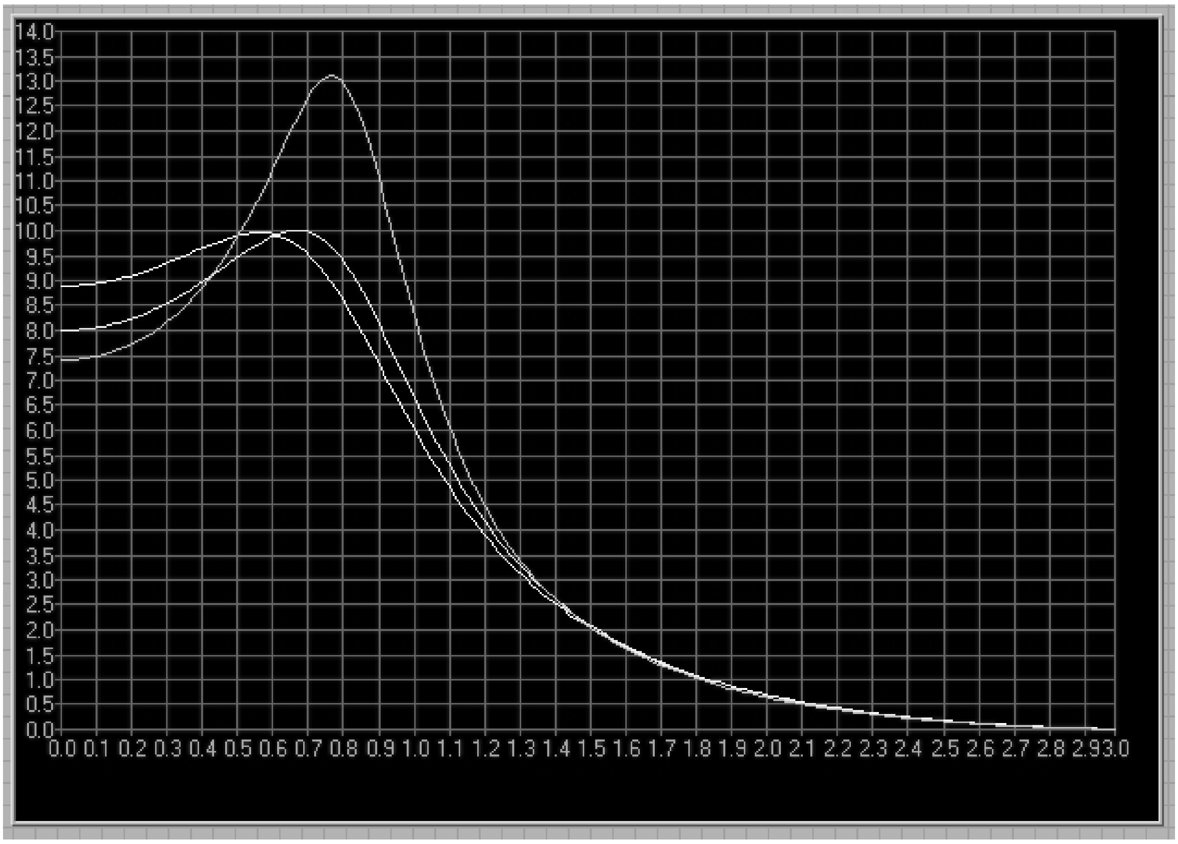
Case (iii) numerator $= z^2 - 1 = (z+1)(z-1)$ => zeros at $z = \pm 1$



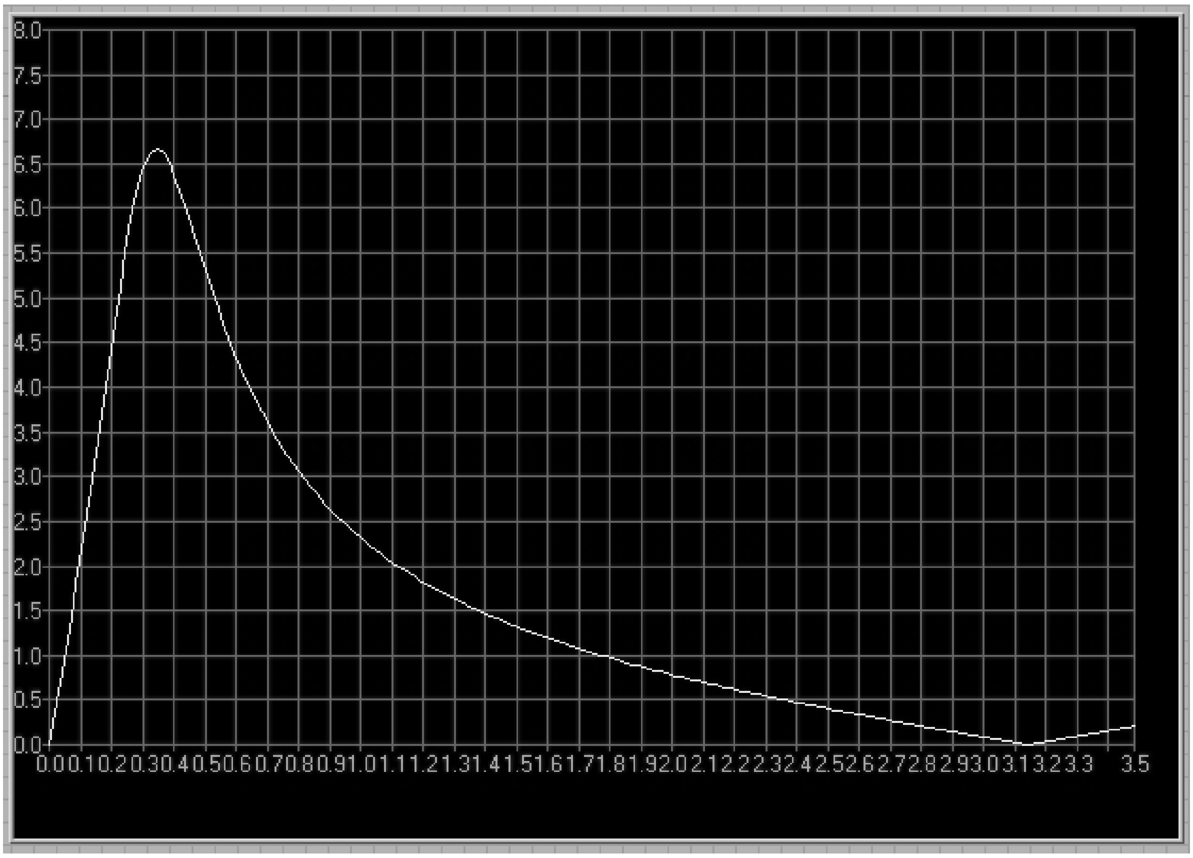
END Q7



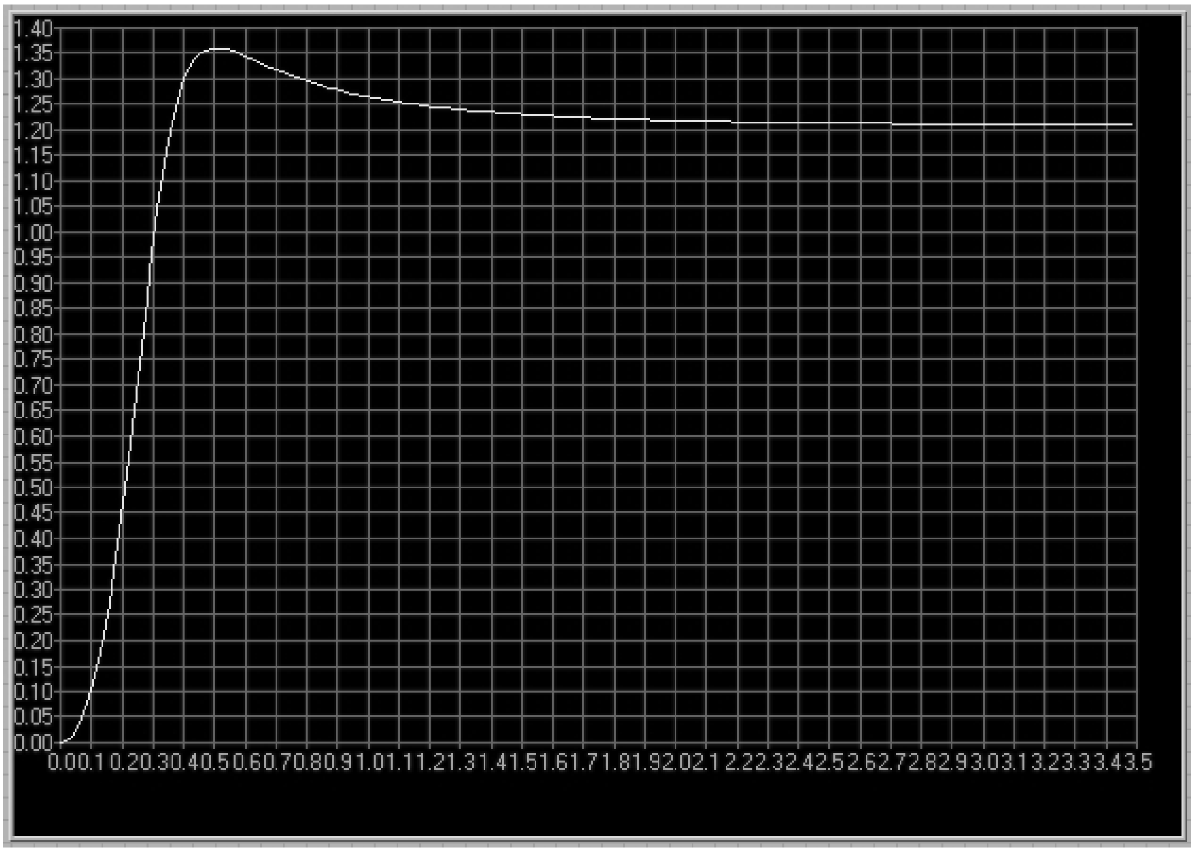
In descending order at origin: f(w)=abs[exp(i2w)+2exp(iw)+1/bexp(i2w)+aexp(iw)+1] for (a,b)= (-1.6,0.81); (-1.1,0.55); (-1, 0.5)



In descending order at origin: f(w)=abs[exp(i2w)+2exp(iw)+1/bexp(i2w)+aexp(iw)+1] for (a,b)= (-1,0.45); (-1,0.5); (-1.1, 0.64)



BPF: f(w)=abs[exp(i2w) + 0.exp(iw)-1/bexp(i2w)+aexp(iw)+1] for a= -1.6, b=0.7



HPF: f(w)=abs[exp(i2w) -2.exp(iw)+1/bexp(i2w)+aexp(iw)+1] for a= -1.6, b=0.7

Question 8
Consider a DT system with sampling rate 20kHz. Obtain estimates of the poles and zeros that realize a lowpass filter with cut-off near 3kHz. Obtain a highpass filter using the same poles.

SIGEx Expt15 p-z in z-plane (IIR)

Prep Solutions

Q8 Sampling rate 20 kHz ⇒ Sampling interval T = 50 μs

Nyquist frq = 10 kHz = 20π krad/s.

To realize a 2nd order LPF we use case (i) in Q7, i.e. two zeros on the real axis at −1.

The 3dB cut-off frq 3 kHz normalizes to $\frac{3\pi}{10} = 0.9425$ rad/s.

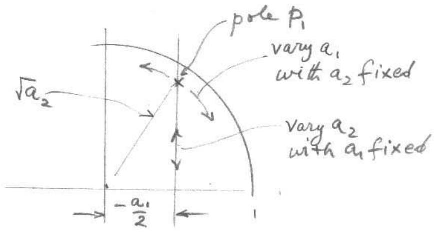
To obtain the required response we will follow a systematic trial and error process whereby the poles are manipulated to progressively "bend" the shape of the response curve in the desired direction. The starting position is not critical, so long as the poles are inside the unit circle, at an angular position around 0.8 of the cutoff. The pole radius should be sufficiently large to produce a moderate overshoot.

The manipulation of the poles with a_1 and a_2 is straightforward. From the results in Q3 we can see the following

(i) with a_2 fixed, a_1 moves the pole along an circular arc concentric with the unit circle.

(ii) with a_1 fixed, a_2 moves the pole so that the real part remains unchanged, i.e. on a vertical line.

Reminders, to avoid migration of the poles to the real axis, check that $a_1^2 < 4a_2$.
For stability poles must remain inside the unit circle.



Refer to the graphs to see how a_1 and a_2 have been varied to bring the response from a rough estimate to a close approximation of the required shape.

END Q8

Question 9

For the same sampling rate as in Question 8 obtain estimates of the poles and zeros that realize a bandpass filter centered near 3.1kHz, with 3dB bandwidth 500Hz. HINT: review Question 7

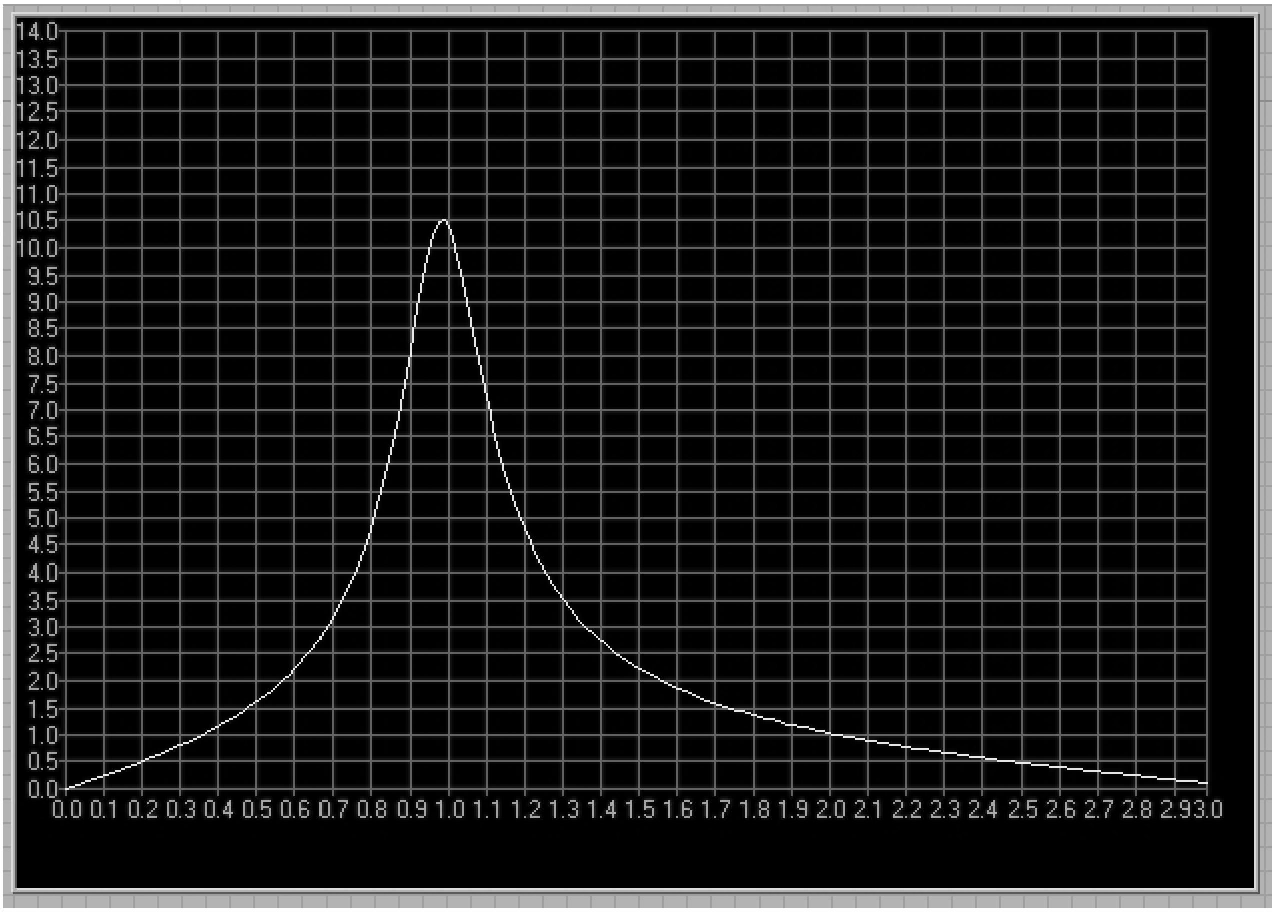
SIGEx Expt 15 p-z in z-plane (IIR)
Prep Solutions

Q.9

Modify the placement of the zeros as in Q7 case (iii) (i.e. zeros at +1 and −1). The bandwidth is controlled with a_2, the center frq with a_1 (refer to Q8).

A fairly close approximation is obtained with $a_1 = -1.0$, $a_2 = 0.81$

END Q9



BPF: f(w)=abs[exp(i2w) – 1 / bexp(i2w) + aexp(iw) + 1] for a= -1, b=0.81

Question 10

Calculate the poles corresponding to these values. Measure and plot the magnitude response at the output of the feedback adder. Note and record the resonance frequency and the bandwidth. Use the poles to graphically predict these parameters; compare with your measurements.

Poles @ 0.8+/-0.5i; hence peak @ 1812Hz. Distance from pole to unit circle = 1-0.95 = 0.05

Estimated Gain at peak = 1/(0.05 x 1.6) = 12; Gain at DC=1/0.54 x 0.54 = 3.45

Question 11

Decrease $|a_1|$ by a small amount (around 5-10%, say) and measure the effect on the resonance frequency and bandwidth. Use this to estimate the migration of the poles. Does this agree with your expectations?

A1=1.4, Fpk=2.3kHz, and BW is constant

Question 12
Repeat step 3 for a 5% decrease of a_2. Compare the effects of varying a_1 and a_2. Which of these controls would you use to tune the resonance frequency? Use the formulas you obtained in the preparation to explain this.

A1 tunes resonantfrequency

A2 controls gain, but affects resonant freq also.

Question 13
With a1 unchanged, gradually increase a_2 and observe the narrowing of the resonance. Continue until you see indications of unstable behaviour. At that point, remove the input signal and observe the output (if needed, increase a_2 a little more). Is it sinusoidal? Measure and record its frequency. Measure a_2. Calculate and plot the pole positions. Note especially whether they are inside or outside the unit circle.

At a2=1.022, the system breaks into self sustaining oscillations, at 10V peak and 2.1kHz.

Using PZPLOT, we find poles at 0.8 +/- 0.62i, with r=1.011 (outside unit circle !)

Frequency of poles according to pole position is 2.094kHz…as measured.

Question 14
Begin with a_2 around -0.9. Describe the effect on the response as the magnitude of a_2 reduces. Measure the frequency of the oscillatory tail of the response and compare with your observations in step 5.

As magnitude of a2 reduces, amplitude of ringing reduces.

Fosc = 1.8kHz, for a2=-0.902

Question 15
In the model of step 14, adjust a_2 to reduce the peaking to a minimum. As well you will need to reduce the amplitude of the input signal to 0.5Vpp to reduce saturation. Confirm this for yourself. Plot the resulting response and measure the new value of a_2. Calculate and plot the new poles. Obtain an estimate of the theoretical magnitude response with these poles and compare this with the measured curve. Why was a_2 used for this rather than a_1?

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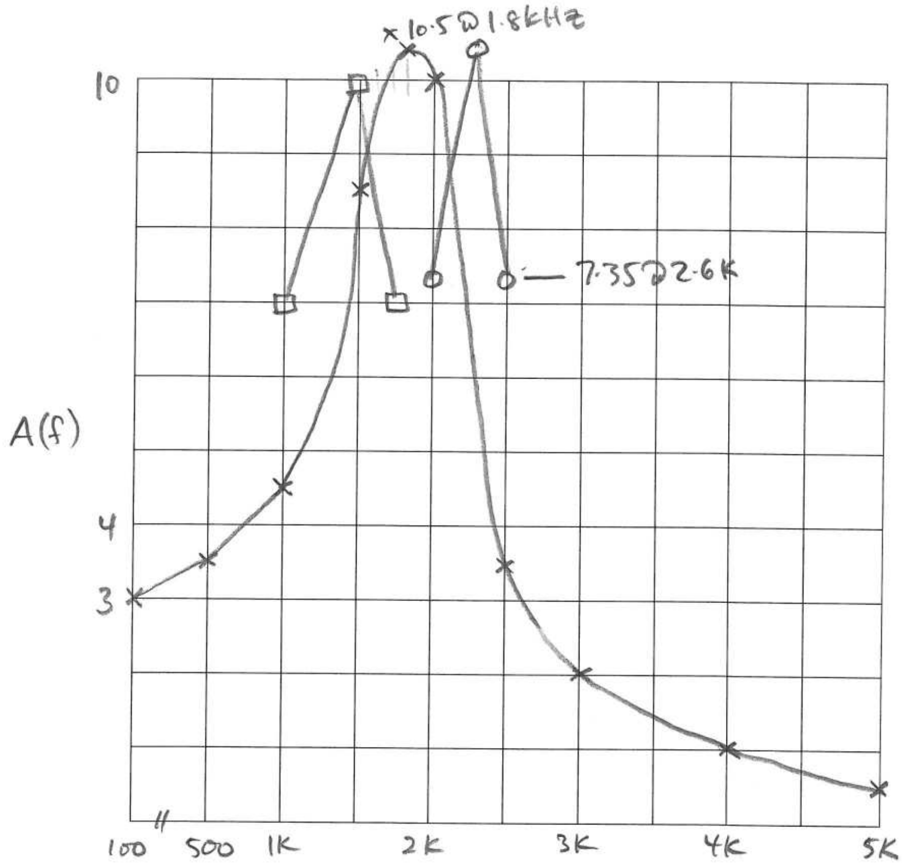


Experiment X 15 : Graph 4

Question 16

Change the polarity of b_1 in the lowpass of step 19 and show that this produces a highpass. Compare with your findings in Question 7.

Question 17
Repeat for case (iii) in Question 7, that is: b_0 = 1, b_1 = 0; b_2 = -1 ; a_0 = 1; a_1 =-1.6; a_2 =0.902; Confirm this is a bandpass filter. Tune a_1 and a_2 to obtain a peak at 3.1 kHz and 3dB bandwidth 500Hz. Measure the resulting a_1 and a_2 and plot the new poles. Compare this with your findings in Question 7.

For ADDER gain settings: 1,0,-1/1,1.1,-0.9, we measure: F-3db at 2.82khz & 3.26kHz, giving approx 400Hz

3dB BW. Other settings will also be suitable.

Question 18
Implement the following case: a_0 = 1, a_1 = 0, a_2 = 0.8, b_0 = 0.8, b_1 = 0, b_2 = 1. Note that b_0=a_2 and b_1=a_1. Measure the magnitude response. Confirm it is allpass. Locate the positions of the poles and zeros. Plot them below for your records.

Zeros: 0 +/- 1.12i; poles : 0+/- 0.89i

Allpass.

Question 19
Change a_1 and b_1 to - 1.6 and confirm the response is still allpass. Examine the behaviour of the phase response. Look for the frequency of most rapid phase variation, and confirm this occurs near a pole. Plot the poles and zeros below for your records.

Zeros: 1 +/- 0.5i; poles: 0.8 +/- 0.4i

Allpass. Pole & zero frequency = 1476 Hz

Question 20
Show your calculation of the where you expect the peak frequency to be using the pole position and sampling frequency.

Poles at 0.8 +/- 0.51. tan θ = 0.51/0.8, hence θ = 32.5 deg., hence f pole = 1806 Hz

Note: this is only a very close estimate, as peak may not align perfectly with pole angle.

Question 21
Confirm this relationship from values displayed on PZ PLOT and show your working here:

A1 = +1.4 & poles @ 0.7 +/- 0.64i

A1 = -2σ = -2 x 0.7 = -1.4 We have setup the ADDER gain as +1.4 (negated)

Question 22
Varying a_2 will vary the gain or peak level of the filter. Notice what happens in the time domain when a_2 = -1.0. The filter breaks into oscillation. View the poles again using PZ PLOT while varying a_2. (Theory states that $a_2 = r^2$).

For a2 = -1, r=1, giving oscill. @ 2050 Hz

Question 23

Confirm that the SIGEx hardware performs as designed by theory in terms of notch positions etc. You will have to use the zero positions mostly in these cases. Why ?

Notches are implemented by placement of zeros on or near the unit circle.

Question 24

Try varying design values and take note of the ORDER of the filter designed. NOTE that the SIGEx experiment we have implemented can only support a 2nd order structure. Note your observations.

4 diff HPF filter designs are available on the DFD TAB.

NB: the input noise spectrum serves as a convenient

multi-frequency signal for viewing the filter responses quickly and easily ie: for

qualitative analysis, rather than quantitative measurements.

Name: ______________________________

Class: ______________________________

X16 - Discrete-time filters - practical applications

Experiment 16 – Discrete-time filters – practical applications

Achievements in this experiment

Pre-requisite work

Question 1

Using the method in Lab15 Question 5, show that the transfer function for the system in Fig.1 is

$$H\_y (z^{-1}) = y/u = (b_0 + b_1.z^{-1} + b_2.z^{-2}) /(1 + a_1.z^{-1} + a_2.z^{-2}) \qquad \text{(Eqn1).}$$

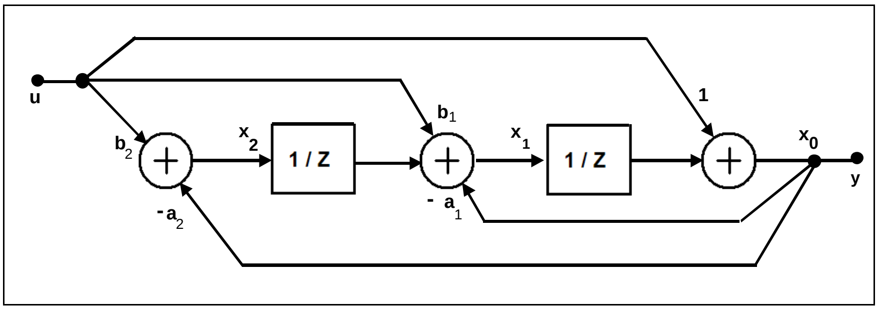


Fig 1: block diagram of 2nd-order Transposed Direct-Form2 feedback structure

Question 2

Consider a filter with a_1 = -1.84 , a_2 = 0.90 , b_0 = 1 , $b_2 = b_0$, b_1 = -1.7. Calculate and plot the zeros of the transfer functions in (Q1).

Question 3

From the results in (Q1) and (Q2) obtain the ratios x_1/y and x_2/y expressed as transfer functions in z. Use these to calculate $|y/x_2|$ and $|y/x_1|$ at the peak of the response of the filter in (Q2).

Question 4

Consider the implementation of the filter in (Q2) using the Direct Form 2 structure in Lab 15 Fig 2. Satisfy yourself using only a quick inspection of the diagram, that with this structure the magnitude responses at the internal nodes are identical. Repeat (Q3) for this case, and compare the outcomes. This comparison will be applied in the Lab, hence it's important to have the analysis ready to use.

Question 5

Consider a transfer function with the coefficients in Question 2 and sampling rate 10ksamples/sec.

(a) Sketch the gain response versus frequency and note the peak and null frequencies. Repeat this with sampling rate 20ksamples/sec. Note that the general shape of the response is virtually unchanged, but the frequency axis has been rescaled.

(b) The outcome in (a) is useful in some applications, however suppose we want to use the faster sampling rate without frequency axis rescaling. This will require relocating the poles and zeros so that their distance from the zero frequency point on the unit circle (1,0) is suitably reduced - by a factor of about 2, in this case. The pole should slide on a line joining

(1,0) and its original position. The zero should remain on the unit circle. Use a computer to plot and compare the new and original responses. Suggest possible adjustments to the poles and zeros to reduce any differences.

Question 6

Look up a suitable reference to confirm that the the bilinear transformations are as follows (T is the sampling interval):

$$s = (2/T)\,.(z - 1)/(z + 1)$$

$$z = (1 + (T/2)s\,)/(1 - (T/2)s\,)$$

These formulas are used to convert continuous time (CT) transfer functions to discrete time (DT), and vice versa. In this exercise we obtain the CT transfer function for the case in Question 2, and reverse the process with a new value of T to produce the DT transfer function for a higher sampling rate.

(a) Find or write a program for implementing the bilinear transformations.

(b) Use this to obtain the transfer function and the poles and zeros corresponding to the increased sampling rate in Question 5. Confirm that the zeros have remained on the unit circle (optional extra: prove theoretically that z plane unit circle zeros always transform to the j axis in the s plane, and vice versa).

(c) Obtain a plot of the gain frequency response with the new sampling rate and compare this with the original and with the approximate case in Question xxx (b).

(d) Compare the positions of the poles and zeros generated with the bilinear transformations versus the approximate case in Question xxx (b).

Question 7

This question is about the effect of errors in coefficient values that may be encountered as a result of limited arithmetic word length. The errors proposed here are of the order that could occur with a 12-bit wordlength.

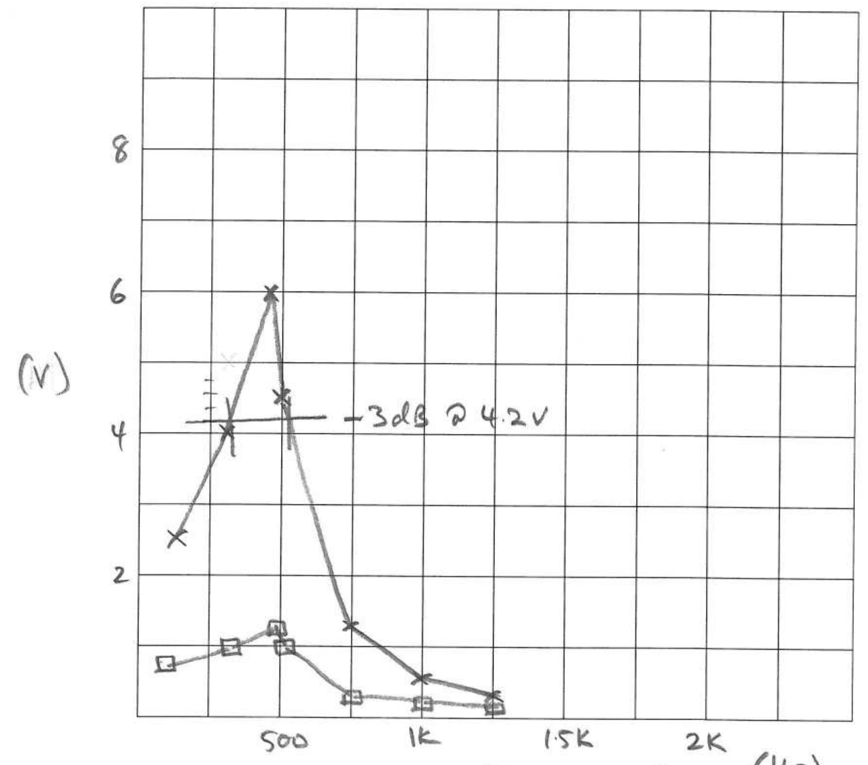
(a) Consider the transfer function obtained in Q.6(c). Change the value of a2 by 0.1 percent. Plot the gain frequency response and compare with the original response.

(b) Repeat (a) for coefficient a1, and then for both coefficients together

(c) Examine the shift in the pole and zero positions for the coefficient errors in (a) and (b). Are these consistent with the gain response errors?

(d) Plot the locus of the movement of a pole as a1 and a2 are varied, respectively. Point to aspects of these loci in the region near the point (1,0) that exacerbate the sensitivity issues relating to coefficient quantization.

(e) Is there any significant advantage with floating point arithmetic compared with fixed point for the effects of coefficient quantization?



Experiment X 16 : Graph 1

X = X_1 & X_2

□ = Y

Graph 1: Magnitude responses

Table 1: Signal magnitudes for Direct form and Transposed Direct form IIR

	Direct Form 2 (for u = 400mv)	Transposed Direct Form 2 (for u = 400mv)	Transposed Direct Form 2 (for u = 1.6V)
Peak (Hz)	412	400	355
u (Vpp)	0.4	0.4	1.6
y (Vpp)	2.6	2.2	11.5
y/u gain	2.6/0.4 = 6	2.2/0.4 = 6	11.5/1.6 = 7
x_1 (Vpp)	12	2.7	11.5
x_2 (Vpp)	12	2.4	9.5
Upper 3dB freq.	500	461	433
Lower 3dB freq.	300	111	216
BW $_{3dB}$	200	350	220

Question 8
What is the maximum level of internal gain you have measured in this filter ?

X30

Question 9
Why is it essential to keep the input signal at a low level ie: 400 mv pp ?

So as not to saturate internal gain stages, especially internal ADDER junction

Question 10
Keeping in mind that the SIGEx circuits maximum signal range is +/- 12V and the maximum gain of ADDER gain stages is +/- 2, what is the maximum level of observable signal you must keep within ?

+/- 6V

Table 2: Implementation table for mapping coefficients

Theoretical value as per block diagram	Implementation label as per patching diagram	Implementation value
	F	1 (fixed)
b_0=1	G	1 (fixed)
b_1=-1.7	B2	-1.7
b_2=1	A2	-1
a_1=-1.84	B0	1.84
a_2=0.9	A0	-0.9
	A1	0
	B1	1

Question 11
What is the difference in internal gain between the non-transposed and transposed structures (in dB) ?

Non-transpose: 12/0.4=30

Transpose: 2.5/0.4 = 6…hence gain difference = 30/6=5 = 14dB

Question 12
Document the transfer function and the poles and zeros for this original filter.

Z: 0.85 +/- 0.53i

P: 0.92 +/- 0.23i

Question 13
What do you expect will happen to the pole and zero positions for a sampling rate of 20,000 samples/sec ?

Nothing. Sampling rate does not influence pole & zero positions

Question 14
What do you expect this filter response to be like with a sampling clock rate of 20,000 samples/sec ?

Peak & null should occur at approx twice the previous freq.

Question 15
What are the -3dB points and bandwidth for this filter at 20,000 samples/sec ?

Fpk = 800Hz. F-3dB = 518 & 975 Hz, BW = 460

Question 16
Approximately how close to the origin will the poles and zeros need to be moved to ?

Halfway

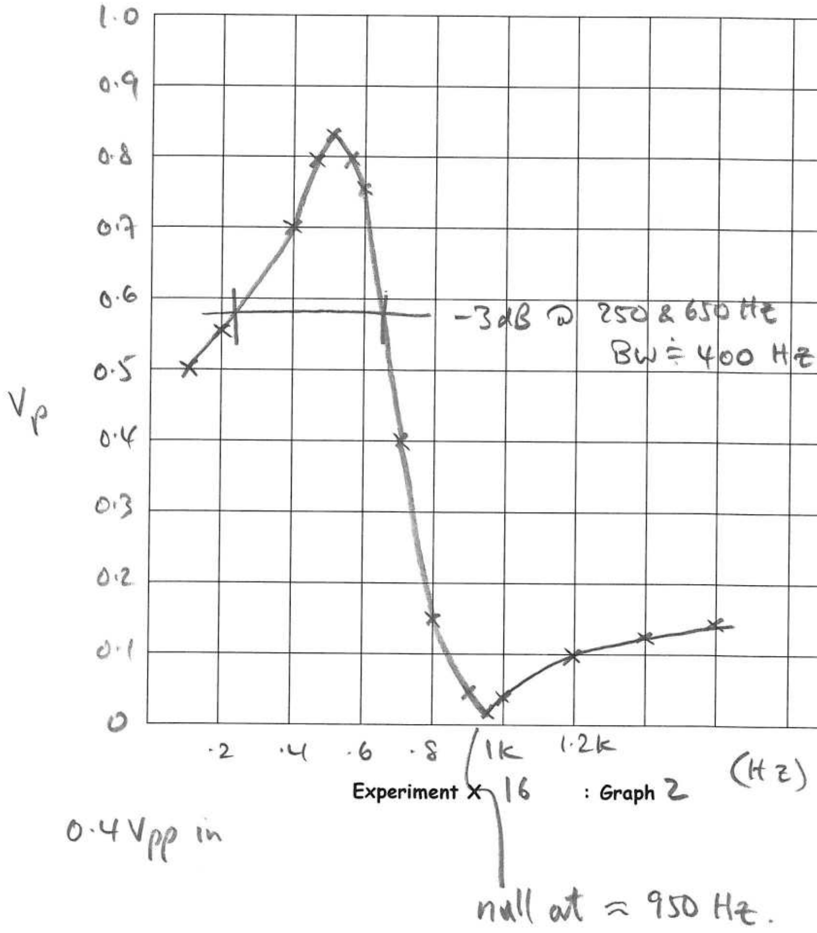
Question 17
What was the best result you were able to achieve in this manner ?

Various results are acceptable. More an exercise to show limits of trial & error.

Question 18
What are the new poles and zeros using the bilateral transformation approach ?
What is the new transfer function for this transformed filter ?
NB: This was covered in the pre-lab preparatory questions.

Coefficients: 1;-1.92;1/1;-1.932;+0.95

Z: 0.96 +/- 0.28i, theta=16.2 deg. Poles: 0.97 +/- 0.13i, r=0.975



Graph 2: Response of new filter at 20kHz

Question 19
What can you say about this new filter in terms of its sensitivity. What are positive and negatives of running this filter design at 20ksamples/sec ?

(-): Higher Q needed, less stable as poles close to unit circle, coefft resolution issues arise.

(+): Easier to filter out output images.

Question 20

Can you suggest a range of angles, in which the poles and zeros would be optimally placed in order to avoid the challenges discovered above ? This may require experimentation or further reading.

Optimum region is θ = 15 – 90 degrees

Emona SIGEx™ Solutions Manual -
Signals & Systems Experiments with the Emona SIGEx
Volume 1

Emona Instruments Pty Ltd
78 Parramatta Road
Camperdown NSW 2050
AUSTRALIA

web: **www.emona-tims.com**
telephone: +61-2-9519-3933
fax: +61-2-9550-1378